Functional Relations
- make them vivid
Research results/ school experiences

Prof. Dr. Astrid Beckmann, University of Education Schwäbisch Gmünd, Germany

Experiences with experiments in lessons

In 8th, 9th grade of all levels (14, 15 years old students)
and 11th grade (16, 17 years old students)
in many classes

Focus: 8th grade students

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Content of this presentation:

- Run of the experiments
- Meaning of non-linear functional relations
- Research questions and some results

Run of the experiments

Run in the classroom:

- two or more double-lessons (about 4 or 5 hours)
- Experiments were offered as stations
- Students work on 2 to 3 or 4 experiments
- partly the students could choose free
- impulses through worksheets
- self-dependent experimenting
- final presentation of one experiment in the plenary

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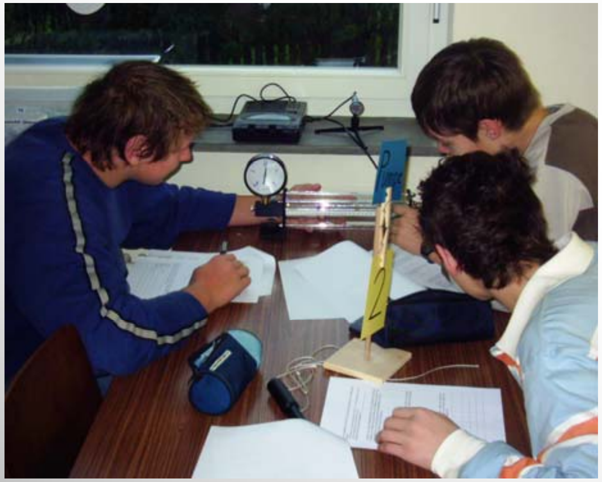

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Meaning of non-linear functions

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Problem: Mathematical lessons stress mainly the linear functions

Meaning of non-linear functional relations

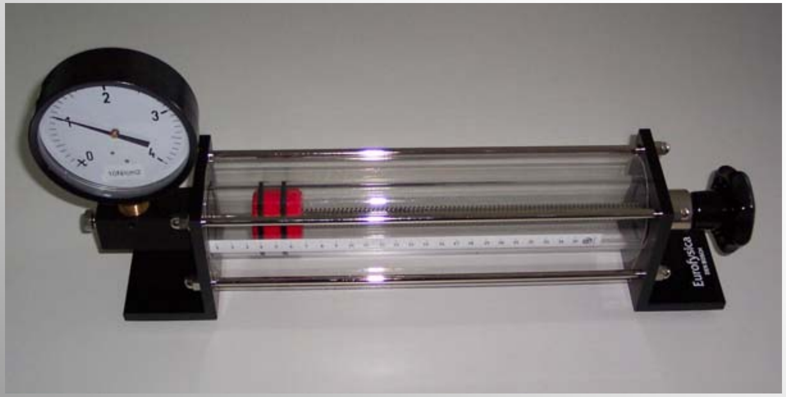
- Stressing linear functions can lead to a shortened view on functional relations

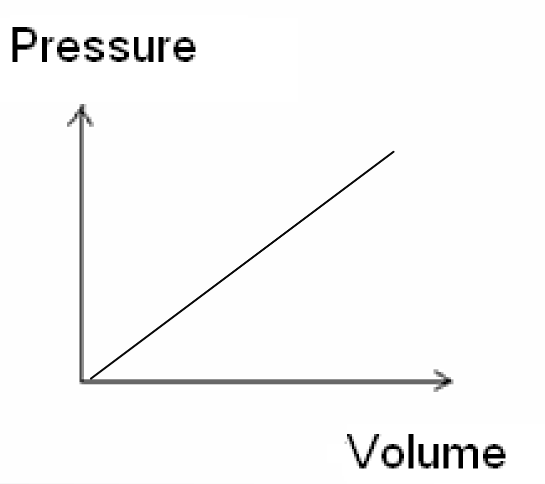
- Experiencing non-linear functions leads to important reflections about functional relations.

Problem

If linear functions characterizes mathematical lessons students think (*wrong*):

All functional relations can be described by a straight line.





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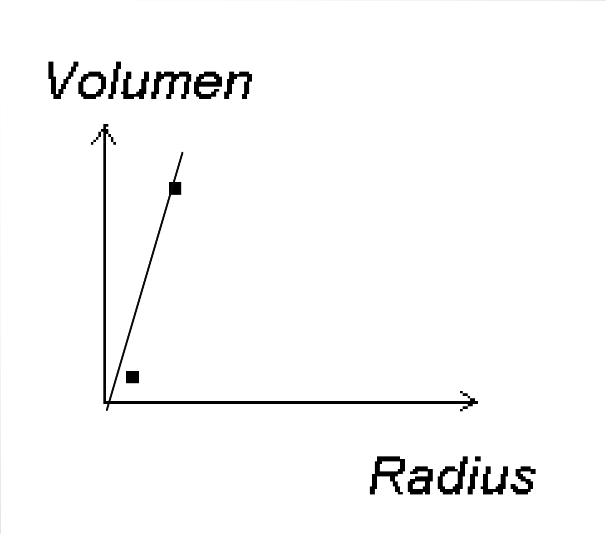
Problem

If linear functions characterizes mathematical lessons students think (*wrong*):

Two points are enough to print the function.



Meaning of non-linear functions

Working with different types of functional relations supports flexible thinking:
- How will the graph continue?
- Which y corresponds to x?
- etc.

Working with different types of functions counter-acts the unreflected drawings of straight lines.

Working with non-linear functions supports a kind of thinking which regards changes and dependences (covariation aspect).

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Research questions and results

Research questions

- Which aspects of the concept of function are touched by experiments?
- Do the student widen their concept understanding?
- Which role does the extra-mathematical context play?
- Which mathematical activities are stimulated?
- Are there further competencies which are supported?

Method of investigation

- Film- and sound-recording,
- Note of short dialogues, episodes
- Evaluation of the working sheets

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Some results

The everyday-life-impulses stimulated argumentations
- also in not very known functional contexts.

Also not so communicative lower achievers talked very vivid about their thoughts.

Example: Experiment Electric Car

Imagine you sit in this car. The car
1.starts at a traffic light.
2.goes around a corner.
3.keeps going straight on on a long highway.
Describe the different movements of the car.
Talk about it in the group.

→ Students discussed the functional relation enactive, iconic and verbal.

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Some results

→ Students discussed the functional relation enactive, iconic and verbal.

Student 1: *That rises* (shows a rising hand movement like the quadratic relation between distance and time).

S1: *Here it moves* (obviously he means the change of velocity in this movement).

S2: (starts the car) *It is going with constant velocity.*

S1: *It should start at the traffic light!*

S2: *Here it moves.*

S1: *It is going quicker.*

S3: (thinks about a better modelling of the real situation) *Or better – on the ground.*

S2: (now on the ground, simulates the start movement at a traffic light).

S1: *It accelerates.*

S2, S3: (delighted and relieved, that they found a characterising description) *jaa!*

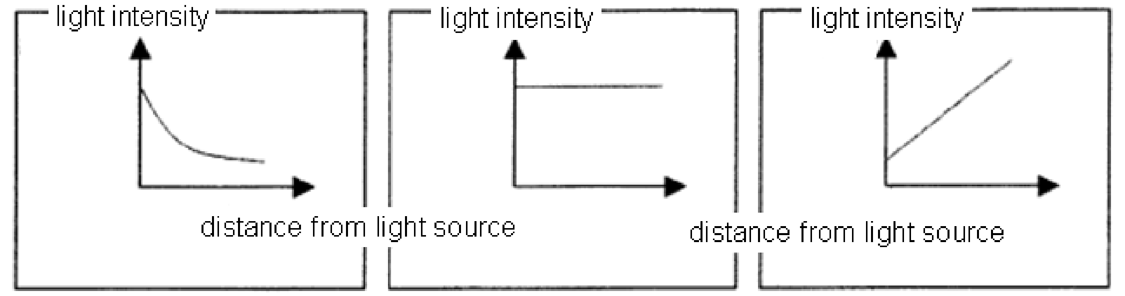
Some results

The students developed an approach according to the method of natural sciences:

Building hypothesis and examining them.

Example 1: Experiment Tunnel

Imagine you are going or driving through a long tunnel.
You cannot see the other end.
How does the light intensity in the tunnel change,
without consideration of the car lights?



→ Students call into question a proportional relation and try to prove it.

Some results

→ Students call into question a proportional relation and try to prove it.

S1: *It is not proportional – normaly.*

S2: *Hey, then let´s do it; then we know.*

S1: (draws a co-ordinate system)

(Together they think about the scale and plot the two measured points)

S2: *It is not proportional.*

S1: *Sure?*

(The students understand that only two points do not answer the question.
Now, they experiment with different tubes systematically
and measure many more points.
Only then they confirm their hypothesis).

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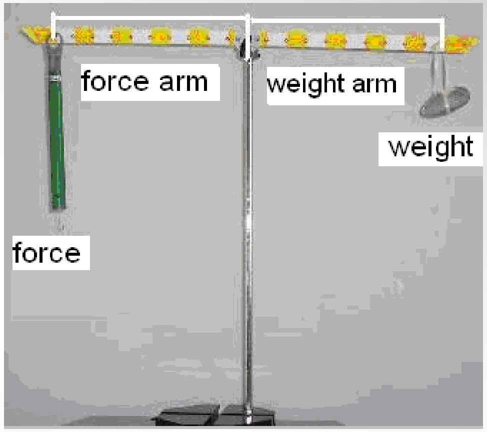
Some results

Example 2: Experiment lever 2



Which relation is there between force and force arm?

→ Students try to predict the next measure point.

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Some results

→ Students try to predict the next measure point.

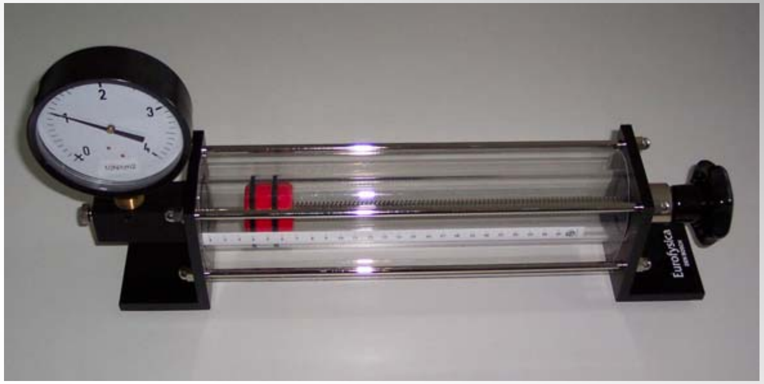
S1:	*Now we hang it at the back.*
S1:	(measures 25,5 cm)
S2:	(measures: 0,6 N) *Actual it should be 36 cm*.
S3:	*It should be one less,* *every time you hang it more to the back.*
S2:	*Now, it is 18.*
S1:	(measures): *That are 1,3 N*.
S3:	*Next should be 0,1 N more.*

Some results

The students show in their discussions thoughts that are typically for functional thinking.

Example 1: Experiment Pressure

Which relation is there
between air volume and air pressure?



From the final presentation in the plenary:

S1: *Then we plotted the points in the co-ordination system.*
The majority thinks it is inverse proportional.
S2: *We think that, because the line goes like that …*
S3: *When air pressure grows, the cylinder shrinks.*

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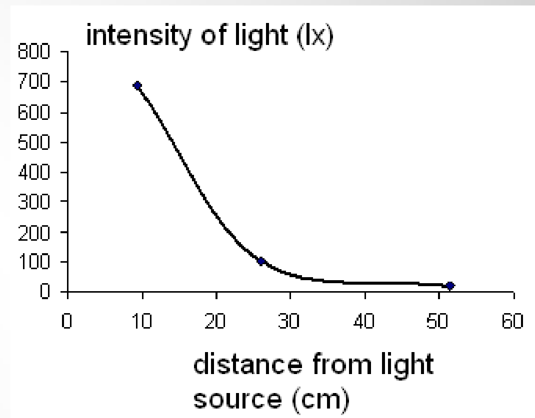
Some results

The students show adequate modelling competencies. In their discussion they use parallel reality, the real model and the mathematical model.

Example: Tunnel



The students use the graph to understand the light situation in the tunnel. They describe correspondences and dependences.

→ So they refer to the **aspects of the concept of function** without any problems in the different models.

Some results

“We called the second project ‘light in a tunnel’.
The further the car drove into the tunnel the darker it got.
The first tube had a length of 9.7 cm.
When we held it to the window, the intensity of the light was approx. 36 lux .
When we held a 30 cm tube to the window, we only measured 0.1 lux.
In the graph we were able to establish exactly
how much light there was at the entrance to the tunnel
and how much there was left at the end of the tunnel.”

Now: Preparing own worksheets for the classroom use

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