Functional Relations
- make them vivid

Prof. Dr. Astrid Beckmann, University of Education Schwäbisch Gmünd, Germany

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Programme

- **Presentation:** Functional Relations and Experiments
- **Trying the material:**
  Functional Relations/ aspects of the concept of function
  - authentical experiences through experiments
- **Discussing the material**
- **Report of research results/ school experiences**
- **Preparing own worksheets for the classroom use**
- **Discussion**

Presentation:

Functional Relations and Experiments

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What is the problem?

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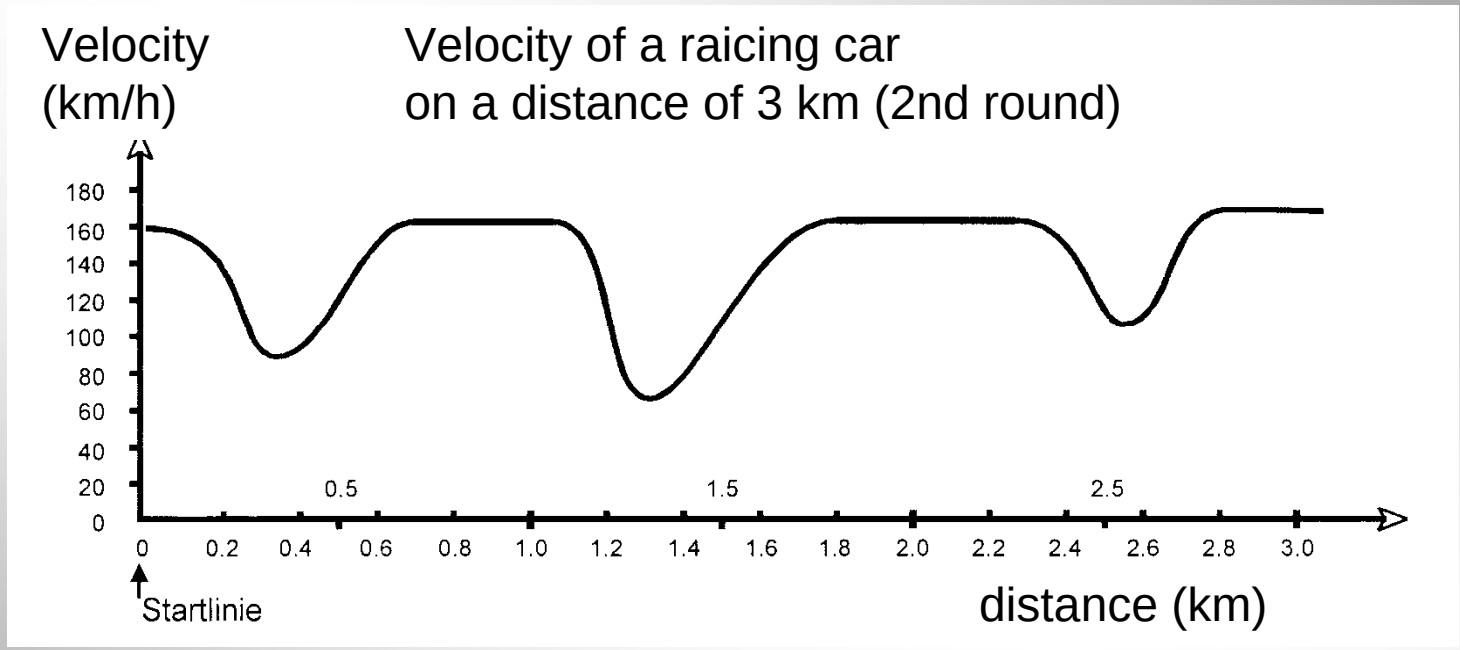

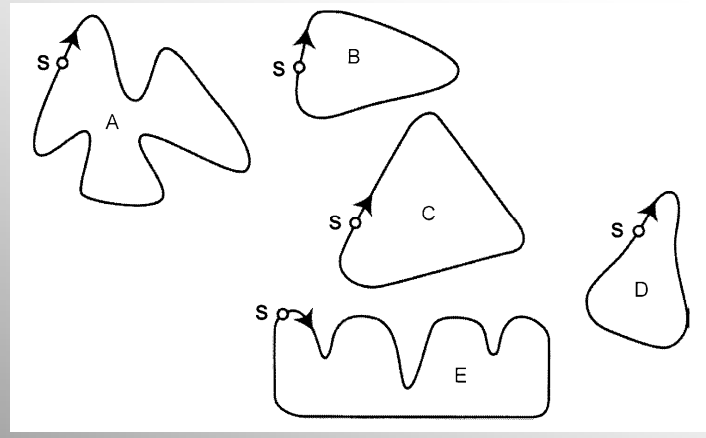

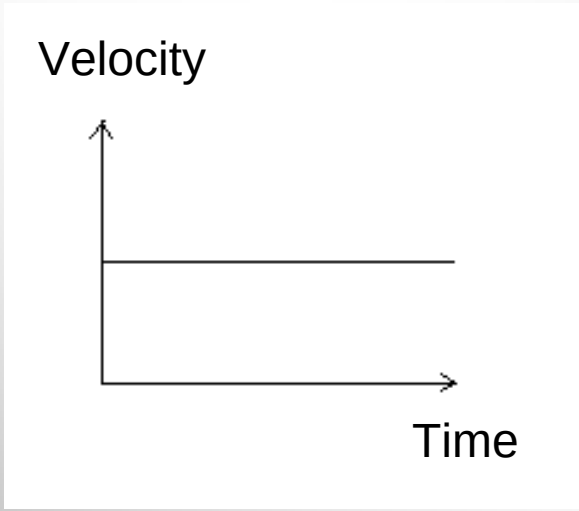

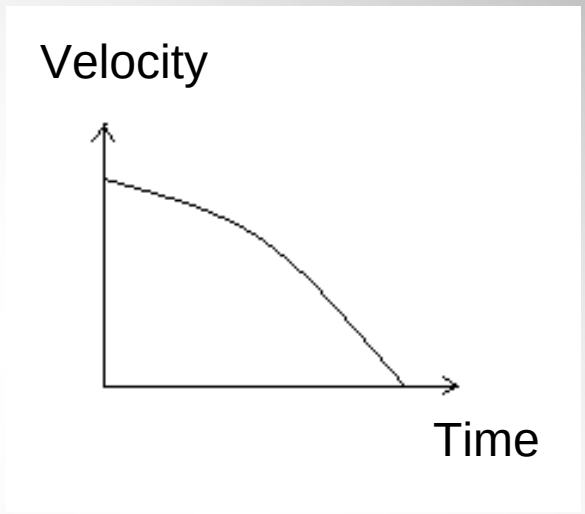

„The car goes straight forward.“

„The car is rolling downhill.“

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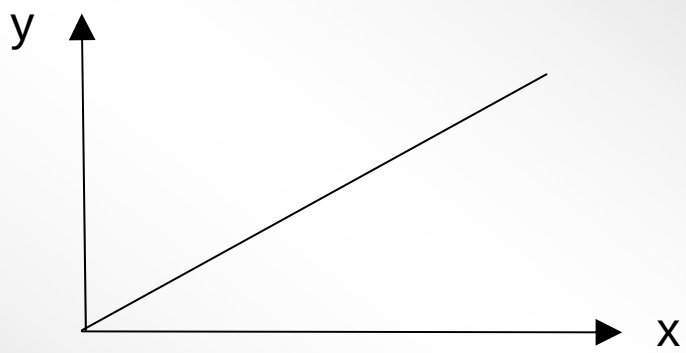
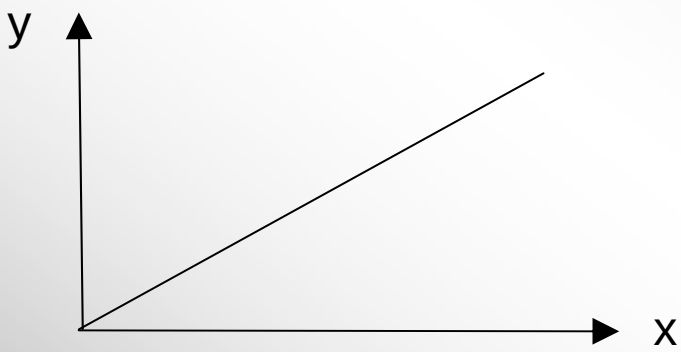
Relation between x and y



„The line devides the co-ordinate system.“

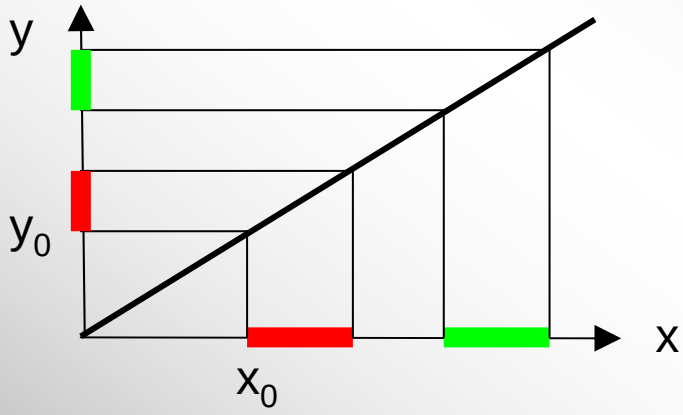
Proportional Function



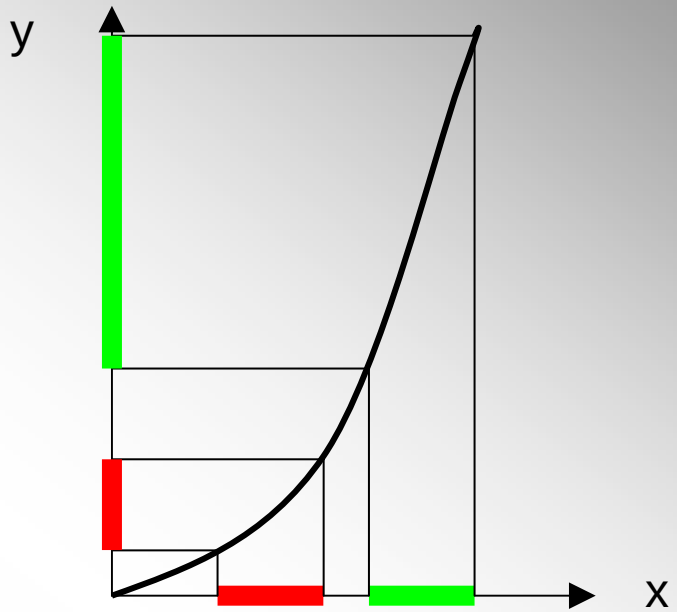
- Straight line through origin
- y/x = constant

- Proportional Function

$$\frac{\Delta y}{\Delta x} = \text{const}$$



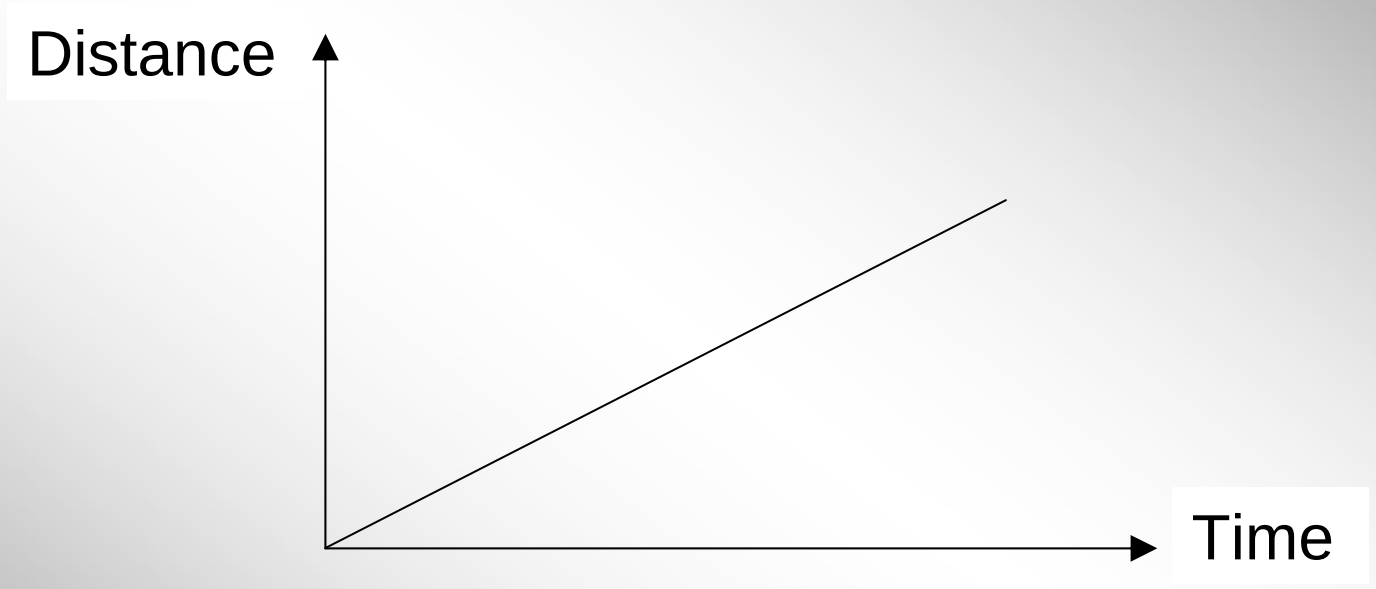
Functional Relation



→ each x corresponds exactly to one y (**Correspondence**)

→ if x changes, y changes as well (**Covariation**)

- How can we support an adequate understanding?



Reality approaches

Authentic Experiences!

→ Experiments

- Connections between aspects of the concept of function and experiments

- **Corresponding (simple): Aquisition of single data**

Dipping a ball
with a special radius

a concrete water volume
will be displaced.

Correspondence: radius – water volume

The simple correspondence is experienced.

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- **Correspondence (continuous):**

Aquisition of different data/ series of measurements

Now: Don´t observe only one ball,
Observe (continuously in a series) more than one.

Each radius corresponds to a special volume.

The continuous correspondence is experienced through activities.

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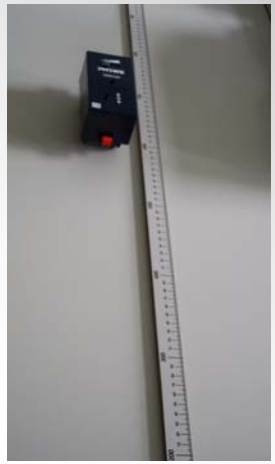
- Covariation (discrete)

Example:
Time and distance, which a car goes in this time.

When we look at our watch at intervals

we notice,
that the driven distance changed as well.



The discrete covariation is experienced.

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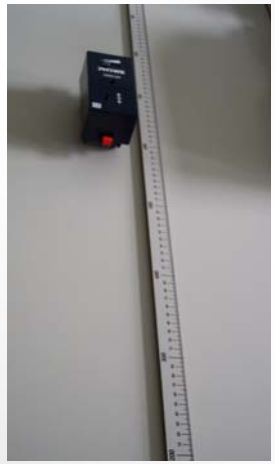
- Covariation (continuous)

Now we do not look at our watch only at intervals, but permanent.
The time goes by and simultaneously we experience that the distance the car goes arises continuously.



The continuous covariation is experienced.

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- Object aspect

To comprehend a function as object means
to comprehend the function as a whole,
which means to be familiar with aspects
like simple and continuous correspondence,
discrete and continuous covariation in all forms of representation,
possible changes and ways of changes.

- Object aspekt

The limits of an experiment motivate to grasp a function like an object.

While experimenting the object aspect is experienced.

„inverse proportional feeling“ while pumping a close air pump.

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- Experiments in the classroom

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Objective: Make the functional relations vivid

That means e.g.:

- Discuss and experience functional relations in reality or in connection with reality
- Experience functional relation authentically
- Make the aspects (correspondence/action and covariation/process) aware – during the single steps of the experiments.

→ Experiments with impulses!

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- Important impulses

for successful experiments in the classroom

- Connection to reality/ Starting impulse:

In leisure parks or fairs you can find a special attraction,
the “Freefall Tower”.
This is a slim tower of iron bars about 50 m high.
The people are first brought up to the top
and then dropped down.

Source: www.pxelquelle.de
ID99300, fotograf: anjume

Do you have experiences with it? What did you feel?

The falling distance could be different at different towers.
What would be the difference between a long and a short fall?

Discuss this in the group. *Find many differences.*

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- Creating hypothesis and proof it

General Task

Describe the relation between quantity (time) and quantity (height).

Verify: Does the relation confirm your presumption?

Describe the special features of the relation.

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- Authentic experiences

Material:
- Ball (Tennis),
- measuring cord,
- stop watches,
- Stairways in the school,
  where the ball can be dropped
  and the falling distances can be measured.

Measure the height and the times,
that the falling ball needs.

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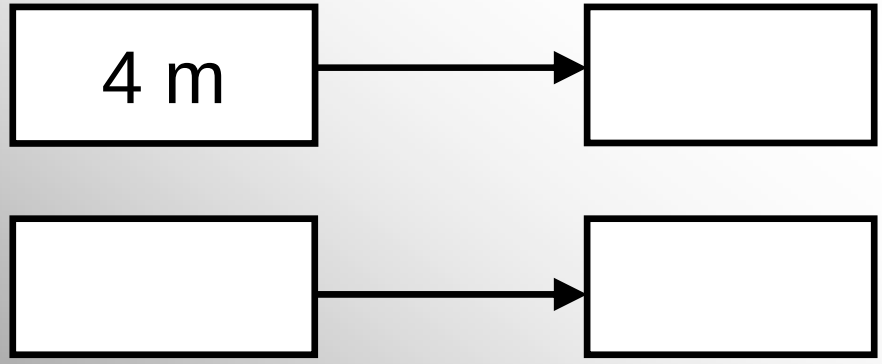
- (simple) correspondence

Drop the ball.
Measure the time for 4 m.

Fill in the data into the right box.



- (Continuous) covariation

Go on measuring.
Fill in the corresponding data

Height in m	Time t in s
0	0
1	1,7
4	3,7
9	5,5
15	7,4

- Covariation

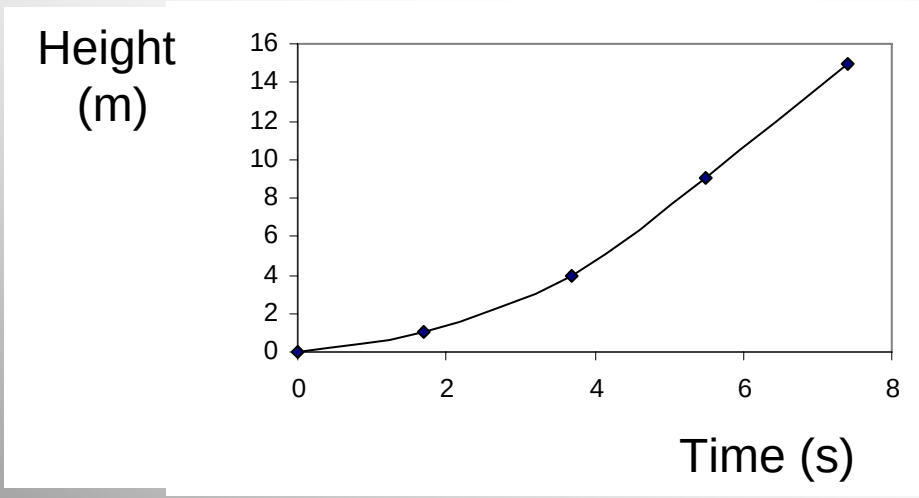
Regard the table.
Are there special relations? ……..

Regard the diagramme.
Describe it…….

- Correspondence

Regard the diagramme.

How long does the ball need for 15 m ?



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- Covariation

How does the time changes
- while dropping the ball 4 m instead of 2 m?
- while dropping it 9 m instead of 7 m?

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- Object aspect

Regard the table.
Is there a calculation rule
for specifying the corresponding quantity
and which is correct for all quantities?
Write it down.

Proof this calculation rule with different measure data.

Draw (with CAS or grafic calculator) the corresponding graph.
Compare it with your data graph.

Experimental steps - in general:

- Get aware with the material
- Which quantities can be changed?
- Select two related quantities: create a hypothesis about their relation.
- Observe the relation: Change x, how changes y?
- Report: y is a function of x
- Measure sytematical
- Evaluate: Which kind of relation? Calculation rule? etc, also qualitative at the graph
- Investigate the relation. Characterize it.
- Use the results for predictions.

- The **ScienceMath** teaching modules

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- In regard to functional relations there exist a lot of experiments!

- Select: according to class, to capacity level, to deepness of interdisciplinarity, to preparation time etc.

$\rightarrow$ different ScienceMath proposals

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Functional relation 1

Simple experiments in mathematical lessons

Functional relations 2

Physical experiments in interdisciplinary lessons

Functional relations 3

„unknown“ functions in interdisciplinary contexts

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Further related modules:

- Proportional factor 1
- Proportional factor 2
- Temperature
- Relationsship between mass and volume of a liquid
- Decaying processes
- Boyle´s Law, Bouancy, Refraction, Thermal expansion and the concept of variable

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Trying the material

Your job:
Work on the worksheet:
Read the text and do the task.
Please reflect continuously:
At which parts which aspects are touched?
Do the students can experience the aspecst of the concept of function?
Where?

Please carry out the experiments!

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