ScienceMath products

Concept of Parallelism
Concept of the Center of Gravity

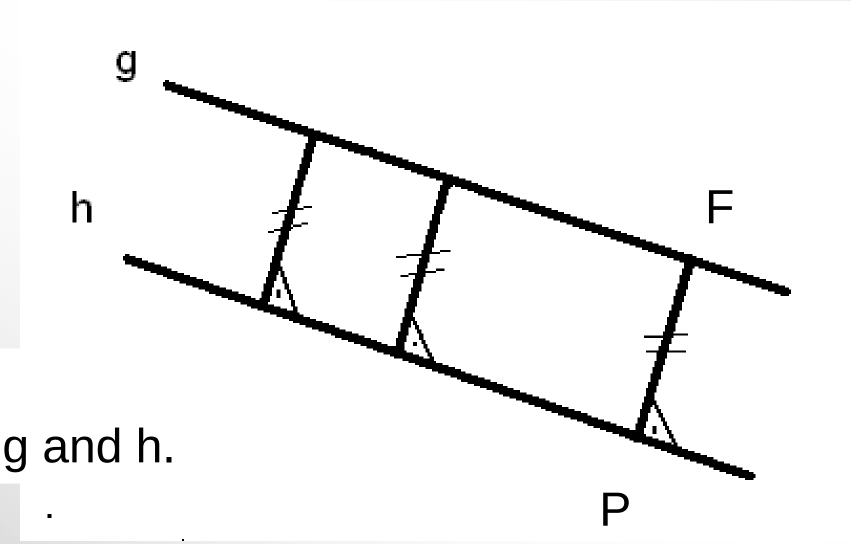
Prof. Dr. Astrid Beckmann, University of Education Schwäbisch Gmünd

Concept of Parallelism

Background

Definition:

Two straight lines g and h are called parallel, if:
The distance of all points of g to h is constant.



Parallel:
equality of distance of the straight lines g and h.

Background

Application oriented:

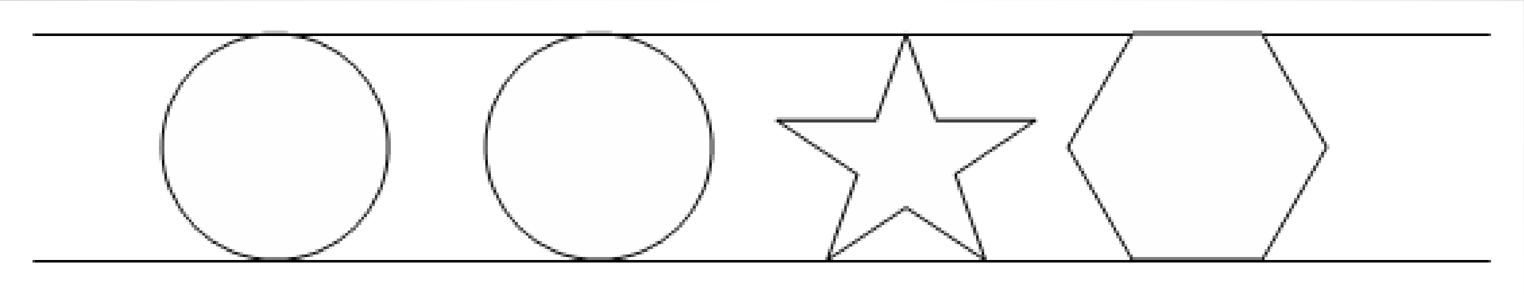
Parallelism can be proven practically by the use of spirit levels.

The position of the board is horizontal,
because the air bubble in the water is seen between the two marks.
The surface of the water adjusts to the (thought) earth level.

Background

Circles between two parallels have to have the same radius.

Objects between two parallels have to have the “same thickness”



Background

Dynamic view:

Do there beside the circle exist further objects
where parallel straights stay parallel under their movement?
How do these objects have to look like
and what features do they have to have?

Material

2 double-rolls “circles” (diameter 12 cm):

Material

1 double-roll “circles with bump” (diameter without bump 12 cm):

Material

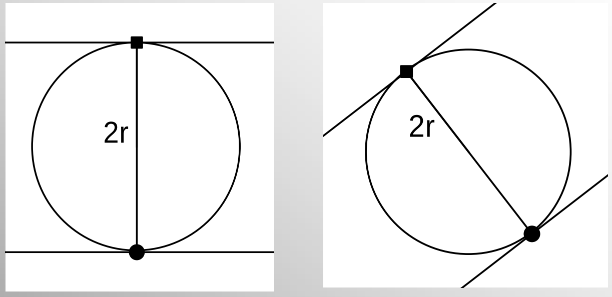
1 or more double-rolls “Orbiforms” (“same-thickness”, corner distance 12 cm):

→ Definition of Orbiforms

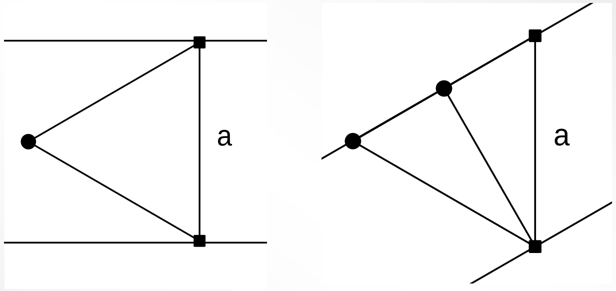
Definition of an „Orbiform“:

An orbiform is a plane with the following feature:
The distance between the points of contact of two parallel and not equal tangents is always the same.

A circle is an orbiform.



An equallateral triangle is <u>no</u> orbiform.



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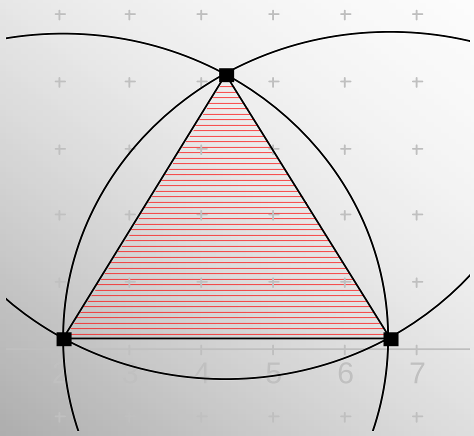
Constructional definition of an “Orbiform”

An orbiform is a plane figure you can construct as follows:



Draw a regular polygon
(equallateral triangle, square, pentagon, and so on).

Draw a circle around one of the corners of the polygon
through two side by side opposite laying vertices.

Repeat same for all corners.

Information:
A circle is an orbiform which results from a square
with arcs with a radius equal to the side length of the square.

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Structure and Performance

In all experimental arrangements
two of the double rolls are placed on a board;
a second board is laid on top.

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Experiment Rolls

Move the upper board over the two chosen rolls
and watch the position of the boards.
Watch also the two spirit levels in four different situations of the rolls.

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Conclusion – wide concept understanding:

While moving the orbiform and the circle, the boards stay parallel.

→ understanding the concept of parallelism through interpretation

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The parallel straight lines stay parallel during the movement

Orbiforms have to have everywhere the same thickness.

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Concept of the Center of Gravity

Trial run: Livinius Fleischer

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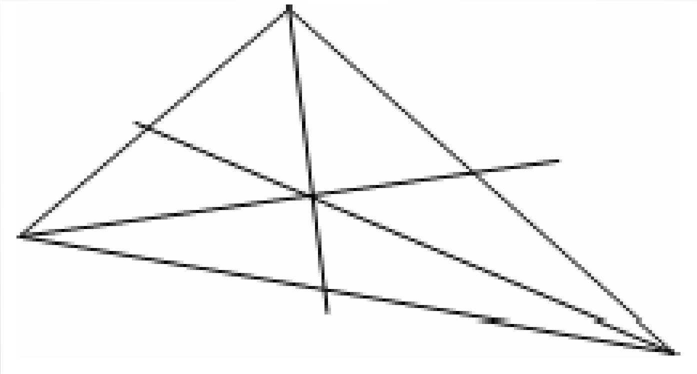
Impulse

Typical topic in mathematical lesson:

Intersection of medians in a triangle



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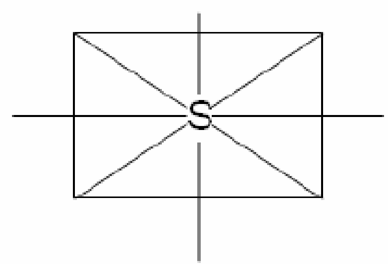
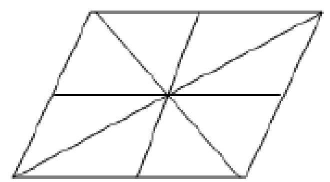
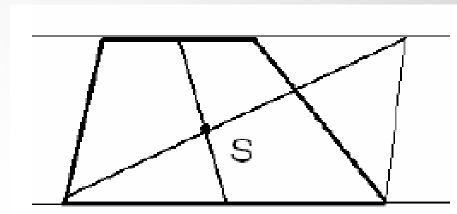
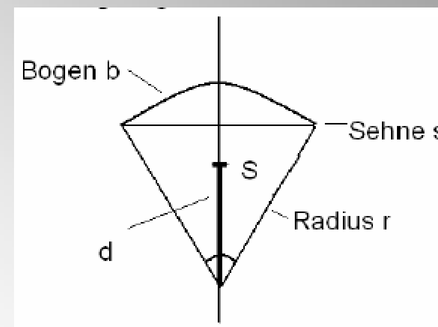
Impulse

Why do not investigate more planes?

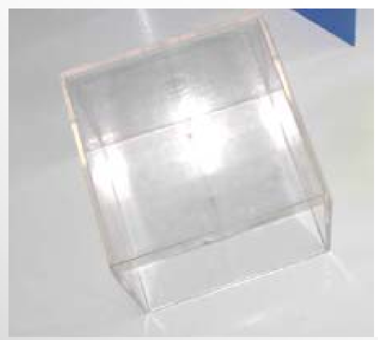








Why do not investigate bodies?



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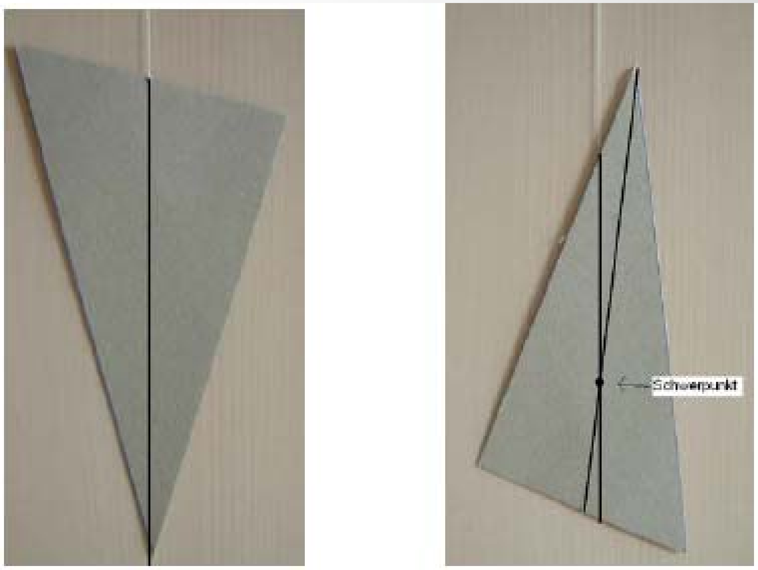
Impulse

Why do not investigate it from a physical point of view?



Why do not support linked learning?

Background

Definition

A body's **centre of gravity**

is the intersection of its lines of gravity.

Line of gravity: line where the force of gravity acts.

Force of gravity: weight F_g of a body

Weight: result of all partial weights

which act on each mass particle

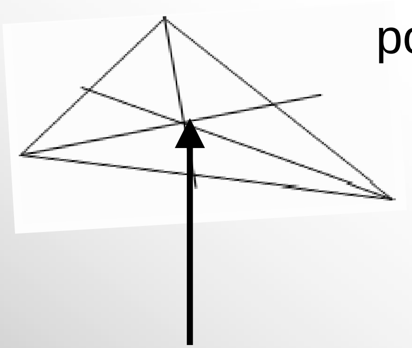
Background

The centre of gravity

Visualisation: Origin of a weight.

Thus, if a weight of equal size but opposite force impacts at that point, the body is in balance.

Example:



point of intersection of the medians

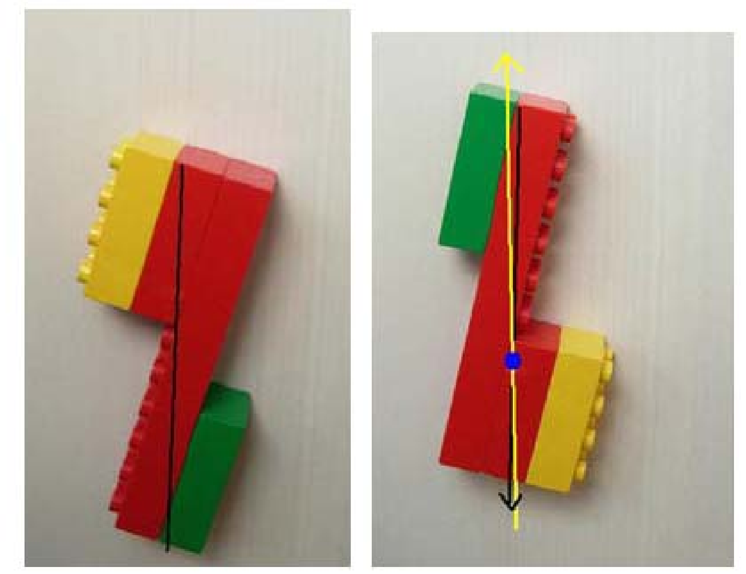
Balance: shore up in the intersection

Background

Methods for determining the centre of gravity

Suspension Method:

A body is hung up in two positions,
and in each the respective lines of gravity are determined. (picture).
Then, the point of intersection is constructed.

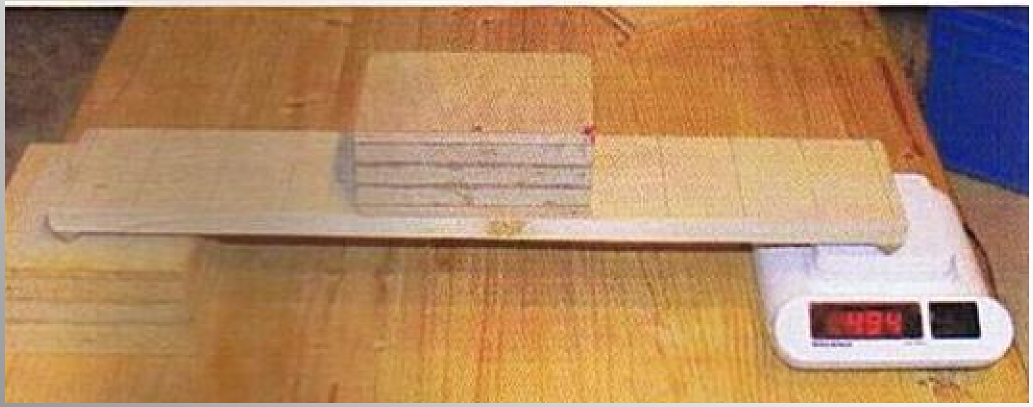


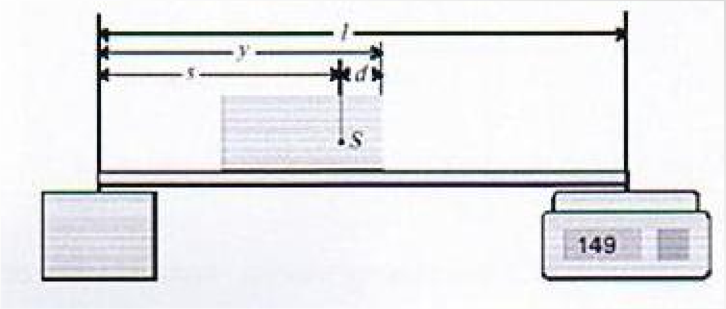
Background

Weighing Method:

According to the ***lever-principle***

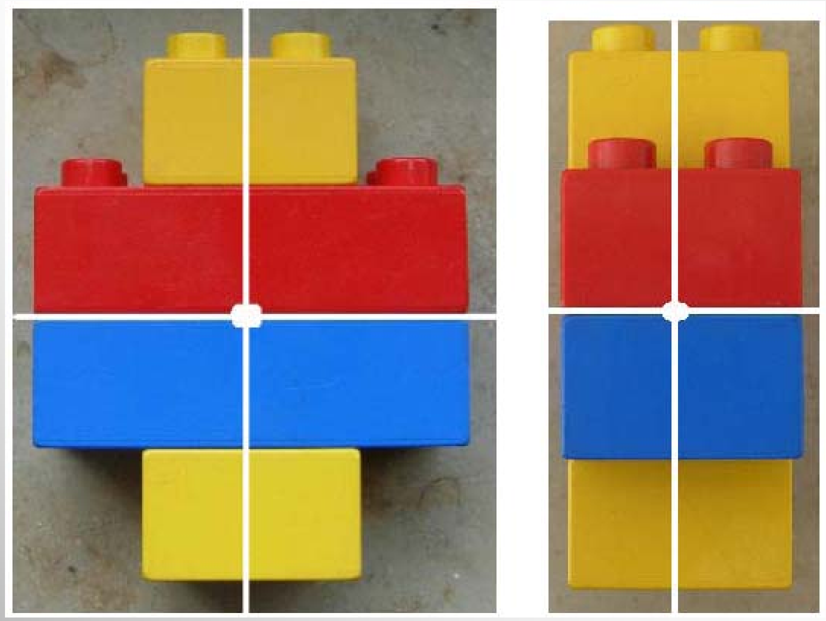
A body is placed in two different positions on a platform, which is placed on a block and on an electronic scale. The lines of gravity are worked out and their point of intersection is determined (picture).





Background

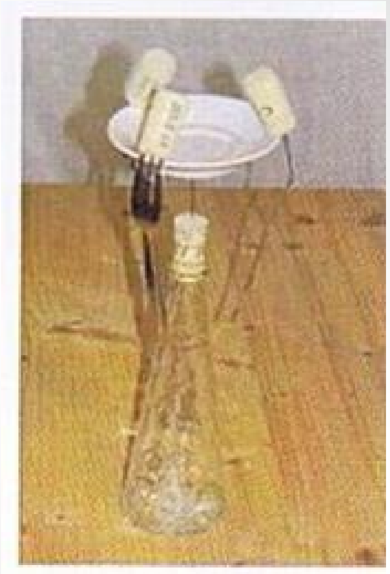
In homogenous bodies **all symmetry lines are gravity lines**



Background

There is a connection between
the center of gravity and the position of equilibrium.

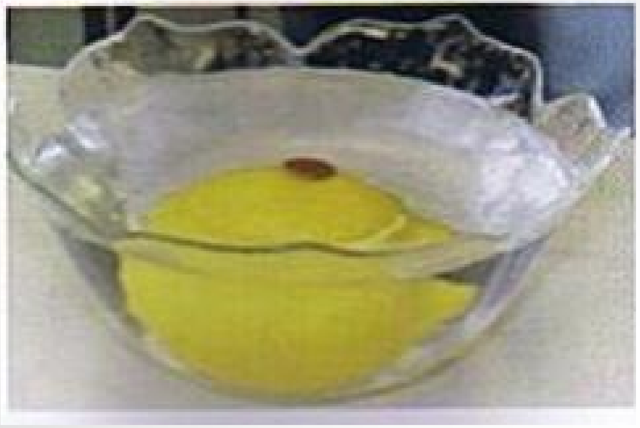
→ Different phenomena



Background

The center of gravity ***can be changed***

e.g. lemon with additional coin



Material

The material is presented in stations.

Stations

station: suspension methods
station: weighing methods
station: symmetry axes
station: planes
station: mathematical definition of points of gravity using a computer
station: carton planes

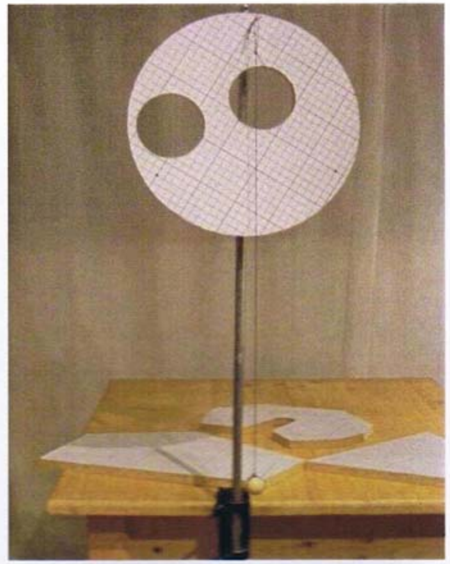
Depending on the students and their level
- All secondary levels are possible

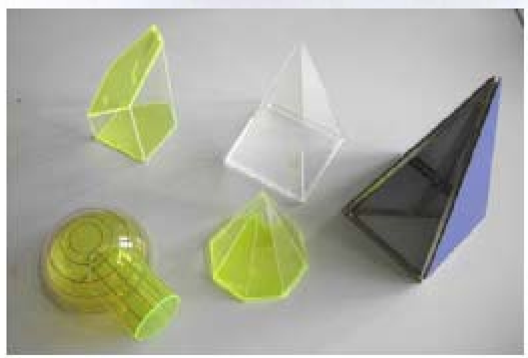
Potential extension, respectively, alternatives for upper secondary level:
Worksheet: centres of gravity in planes and bodies

Potential extension:
Connection between stability and position of centre of gravity
Stations: Phenomena

Material

There are many ideas!

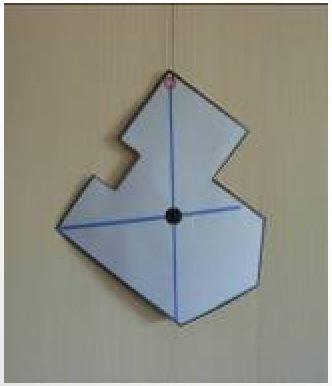




Material

Station: Planes 1

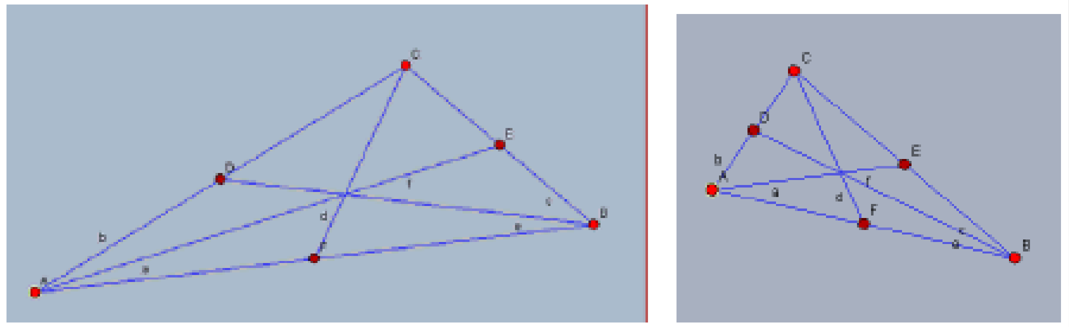
The cardboard planes are hung up in various positions,
the lines of gravity are marked in each
and the centre of gravity is determined by the point of intersection
of the lines of gravity.



Material

Station:

Determining centre of gravity with help of a computer
(Using a dynamic geometry system)



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Material

Station: Planes 2

The centre of gravity of an area **is first worked out graphically**
(possibly by experiment as well)
on the basis of the insights gained in the previous experiments.

Its position is determined and described by the size of the areas.
For example: The centre of gravity in a triangle is the intersection of the medians.

Its correctness is checked
by hanging the body attached to this point or supporting it there.

Material

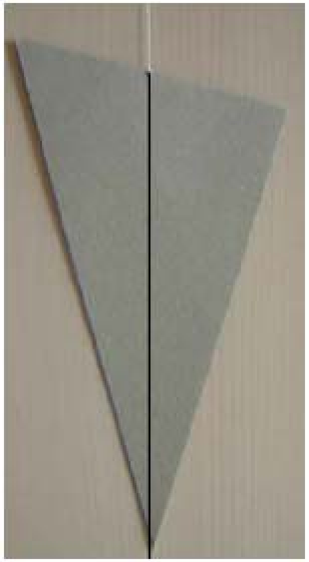
Potential extensions:

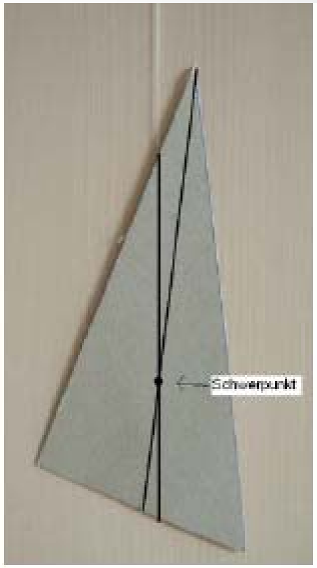
- Phenomena
- Analytical tasks

Station:

General Task:
Establish the position of the centre of gravity of a triangle, rectangle, cube, pyramid etc.

Task 1: Centre of gravity in a triangle





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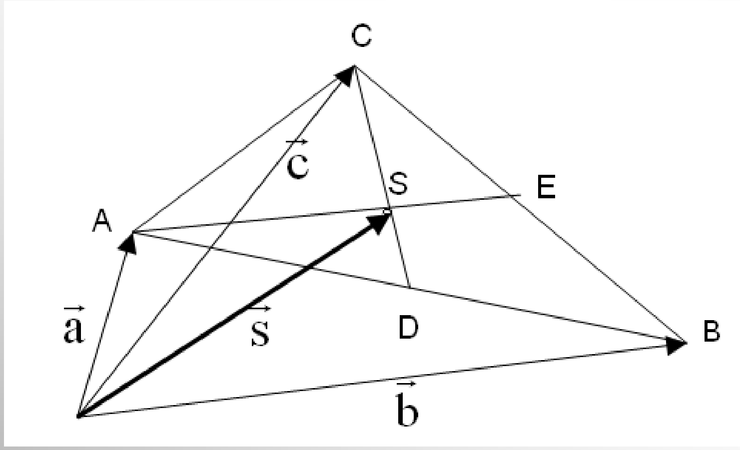
Task 2: Analytical Description of the Centre of Gravity of a Triangle



$$\vec{s} = \vec{a} + \overrightarrow{AC} + \frac{2}{3}\overrightarrow{CD}$$

$$= \vec{a} + (-\vec{a} + \vec{c}) + \frac{2}{3}(-\vec{c} + \vec{b} - \frac{1}{2}\overrightarrow{AB})$$

$$= \vec{c} + \frac{2}{3}(-\vec{c} + \vec{b} - \frac{1}{2}(-\vec{a} + \vec{b}))$$

$$= \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$$

Task 3: Centre of Gravity of a Quadrangle

$$\vec{s} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c} + \vec{d})$$

Right or not right?

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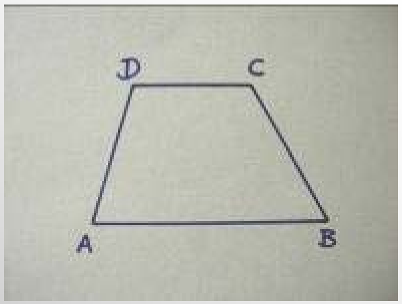
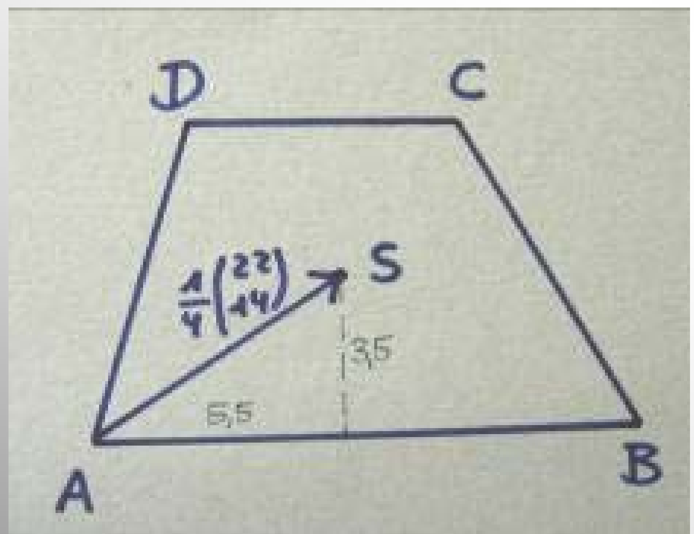
Task 3: Centre of Gravity of a Quadrangle

$$\vec{s} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c} + \vec{d})$$

Right or not right?



$$\vec{s} = \frac{1}{4}\left(\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}12\\0\end{pmatrix} + \begin{pmatrix}8\\7\end{pmatrix} + \begin{pmatrix}2\\7\end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix}22\\14\end{pmatrix} = \begin{pmatrix}5,5\\3,5\end{pmatrix}$$



Seems to be right, but….

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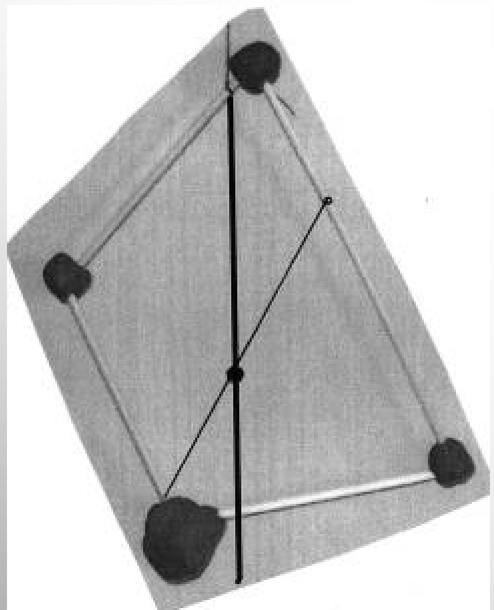
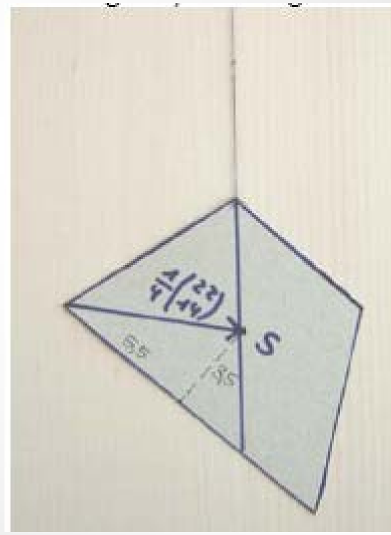
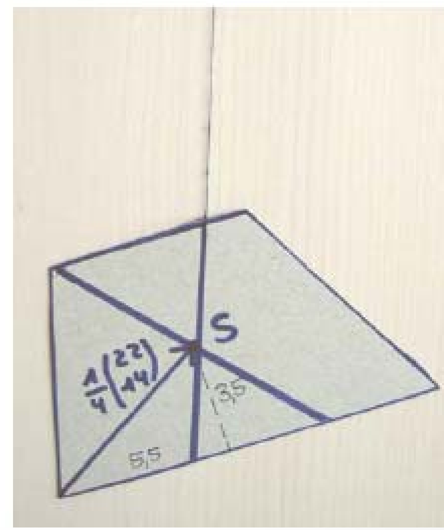
Task 3: Centre of Gravity of a Quadrangle

Experimental check leads to different centres of gravity



(4.7/4.0)





(5,5/3,5)

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Task 3: Centre of Gravity of a Quadrangle

The physical centre of gravity is given through the following equation:

$$\vec{s} = \frac{1}{m_1 + \ldots + m_k} \sum_{i=1}^{k} m_i \overrightarrow{x_i}$$

m_i indicates the mass of the body point i

x_i its positions.

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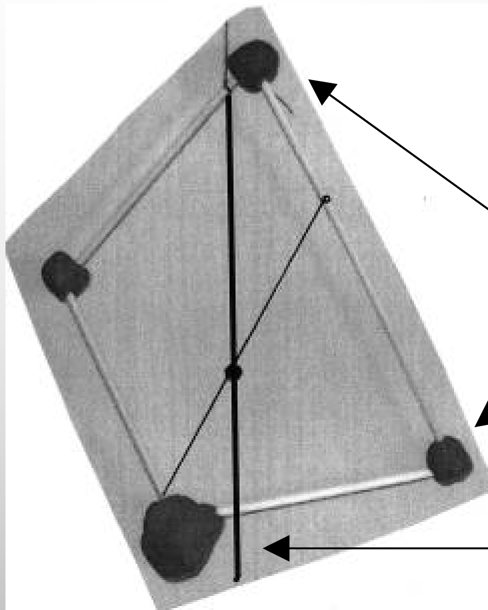
Task 3: Centre of Gravity of a Quadrangle



→ Centre of gravity depends on the mass distribution

0,1 g

0,2 g

$$\vec{s} = \frac{1}{0{,}1+0{,}1+0{,}1+0{,}2}\left(0{,}1\begin{pmatrix}0\\0\end{pmatrix}+0{,}1\begin{pmatrix}12\\0\end{pmatrix}+0{,}1\begin{pmatrix}8\\7\end{pmatrix}+0{,}2\begin{pmatrix}2\\7\end{pmatrix}\right)$$

$$= \frac{1}{0{,}5}\left(\begin{pmatrix}1{,}2\\0\end{pmatrix}+\begin{pmatrix}0{,}8\\0{,}7\end{pmatrix}+\begin{pmatrix}0{,}4\\1{,}4\end{pmatrix}\right) = 2\cdot\begin{pmatrix}2{,}4\\2{,}1\end{pmatrix} = \begin{pmatrix}4{,}8\\4{,}2\end{pmatrix}$$

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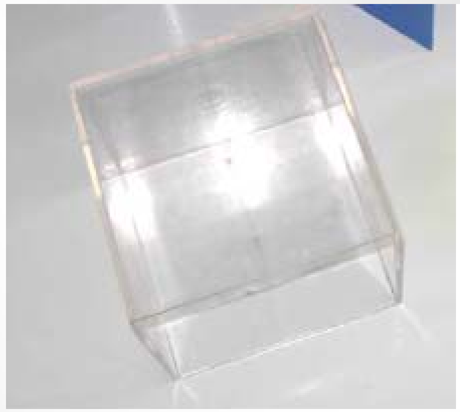
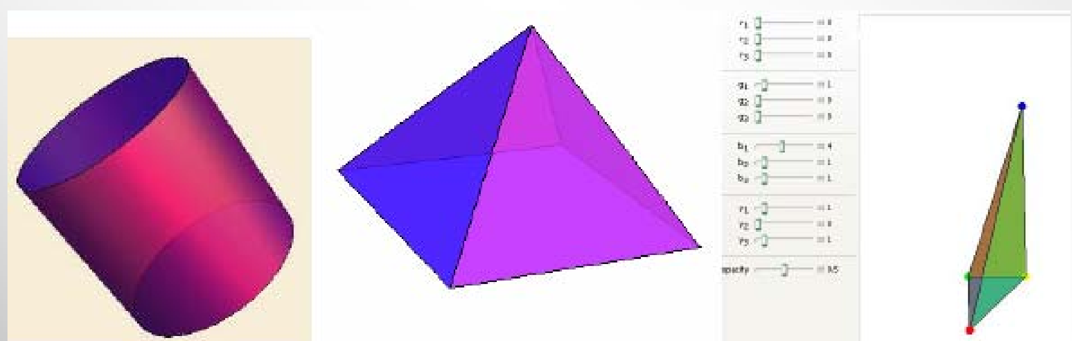
Task 4: Centre of Gravity of Bodies



E.g. Analytical and experimental investigations of Solid and hollow pyramides etc.

Check at computer



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For concrete worksheets, solutions and more ideas

please visit

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