Students' discussions about mathematics and society

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Math Literacy and reflective citizenship

- Mathematical Literacy:
  - ... is an individual's capacity to identify and **understand the role** that mathematics plays in the world, **to make well-founded judgements** and to use and engage with mathematics in ways that meet the needs of that individual's life **as a constructive, concerned and reflective citizen** (OECD, 2006, p. 72).

- Relates to the action-notion of 'competence'
  - to have some knowledge and to be able (and willing) to apply that knowledge to make informed judgements or decisions.

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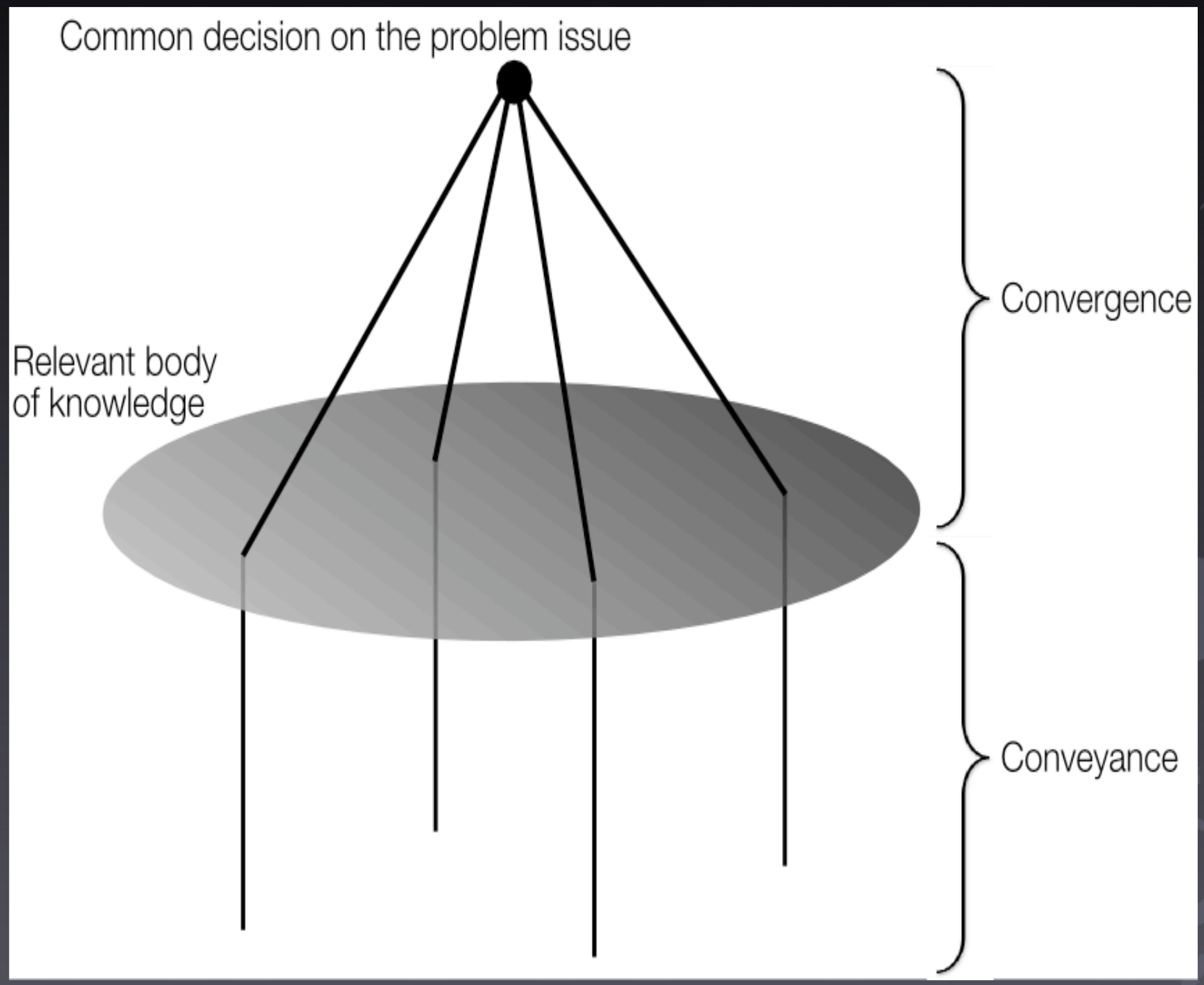
Focus: Convergence processes



Convergence: “the discussion of preprocessed information about each individual’s interpretation of a situation” where the “objective is to agree on the meaning of the information, which requires individuals to reach a common understanding and to mutually agree that they have this understanding”

Conveyance: “the transmission of a diversity of new information […] to enable the receiver to create and revise a mental model of the situation”,

(Dennis, Fuller, & Valacich, 2008, p. 580)

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Purpose

- Consider activities in which groups of students **discuss** a socio-mathematical issue (e.g. population growth, pandemics, gambling, traffic problems etc.) and make **group decisions** about that issue.
- There is an interesting interdependence of
  - Interdisciplinarity
  - Modelling
  - Socio-mathematical decision-making
- Aid in the theoretical analysis when designing teaching materials:
  - Sensible problem issues on which the students need to make a decision can guide students' learning trajectories (≋paths) and thus structure the disciplinary concept field involved.

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First problem from science/math education

- **The problem of relevance:** ‘Traditional’ school science paints a picture of science as being unpopular/boring and irrelevant (European Commission, 2004).

- A variation of the problem of relevance is the **problem of isolation** (well-known in mathematics education)
  - ‘Traditional’ school math focusses on specialised algebraic techniques and does not necessarily prepare students for reflective application of math in other subjects (indeed for critical citizenship in general). (Michelsen, 2006b)

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Second problem from science/math education

- **The problem of operationalization:** Call for more teaching materials and approaches to science teaching that involves the students' capacity to operationalize their scientific knowledge on societal issues (cf. e.g. Sadler, 2004; Zeidler et al., 2005)

- A variation of the problem of operationalization is the **problem of domain specificity** (Niss, 1999):
  - When a concept is introduced in a narrow mathematical domain, the student may see it as a formal object with arbitrary rules. This results in the recognised difficulty of application of the concept in new settings (Michelsen, 2006b; Michelsen & Nielsen, in press).

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A solution to the two problems?

- putting students in situations where they have to **discuss** a socio-mathematical issue and make **group decisions** about that issue.
- Notice:
  - Socio-mathematical issue $=_{def}$ a problem which is of a ethical/ political/societal nature that relates to mathematical content. (That means that an informed decision on the problem requires using mathematical knowledge and/or methods; and often also science)
- → Socio-mathematical issues are **interdisciplinary**

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But....

- Then we meet a new problem!!!
- Simply involving students in discussion activities is no guarantee that they will use mathematics to make the decision.
- And, on the other hand, if we - as teachers - plan too much on before hand we take away the autonomy of the students.
- Dilemma:
  - the Scylla of radical student autonomy (no anchoring, and no mathematics used to organise the problem issue)
  - the Charybdis of no student autonomy (full pre-defined structure, the students visit “monuments” instead of finding things out)

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Horizontal Linking & Vertical Structuring

- We can understand what is at play in interdisciplinary and socio-mathematical activities by looking at Freudenthal's (1991) distinction between:
  - Horizontal processes (identification of subject specific tools/ concepts that can organize/solve a problem outside the subject)
  - Vertical processes (reorganisation of the tools/concepts within the idiosyncratic system of tools/concepts in the subject)

- Socio-mathematical activities involve two processes:
  - Horizontal linking: mathematization of extramathematical issues
  - Vertical structuring: conceptual anchoring in the systematic framework of mathematics.

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Basic argument

- A teleological restraint on horizontal linking - socio-mathematical decision-making - can **guide** the theoretical trajectory of students' reorganisation in vertical processes.

- Modelling competencies are interdisciplinary competencies
- Decision-making competencies rely on modelling competencies
- Trajectories of student's modelling activities can be guided by specific aims.

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Decision-Making Processes I

- We are *not just* interested in training them to be deciders, *but also* active agents.
- The actions we consider here are *intentional actions* that are performed wilfully. (Coerced or automated behaviour is certainly not critical activity). (see e.g. Davidson, 1980)
- (Mogensen, 1995; Breitling et al., 1999): Being a competent actor involves components in four dimensions:
  - **Cognitive** (knowledge about the issue, knowledge about handling the issue)
  - **Social** (knowledge about the possibilities within a community)
  - **Personal** (being intent-full, feeling responsible)
  - **Value** (identifying normative criteria)

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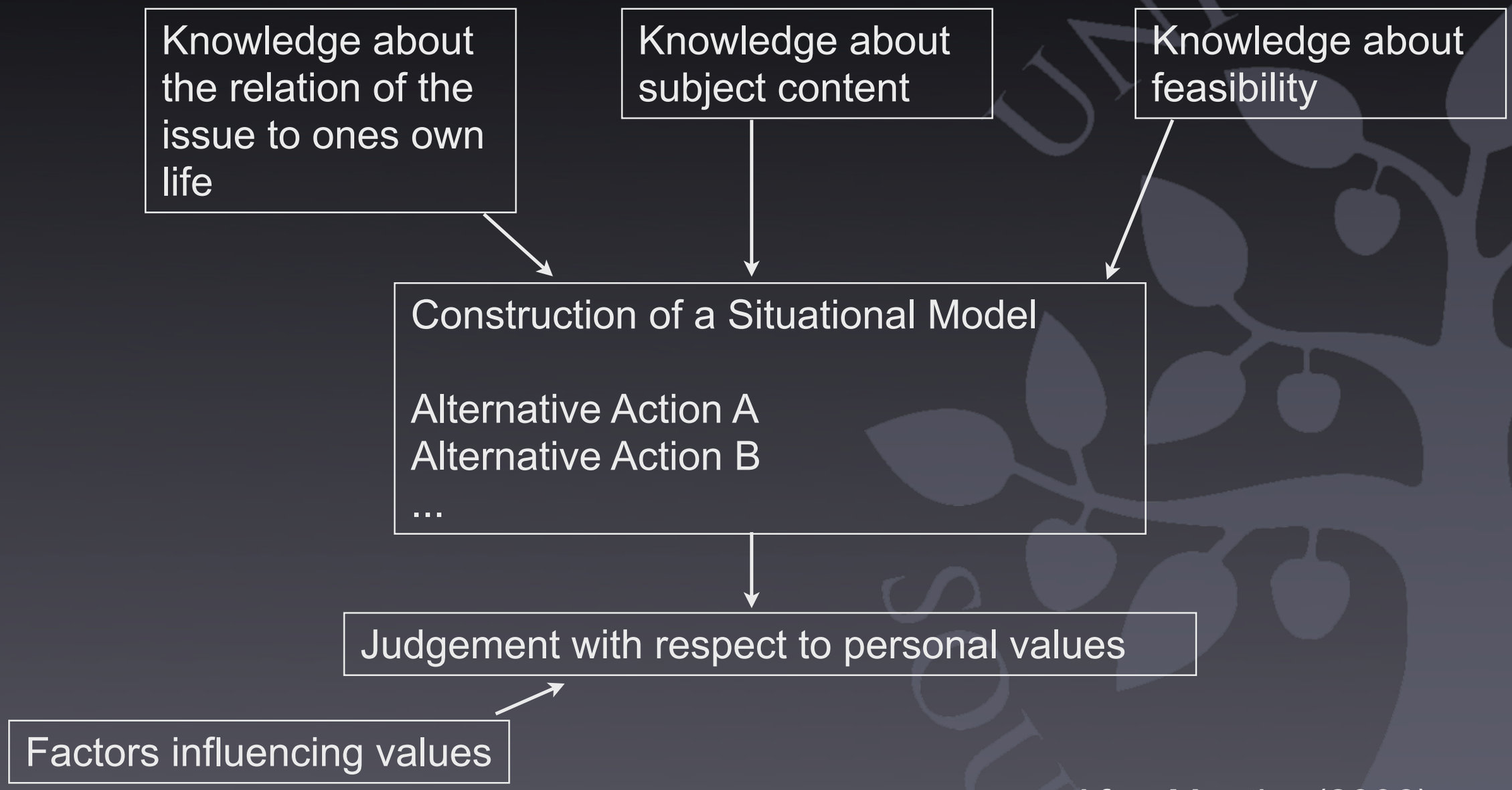
Decision-Making Processes II



After Menthe (2006)

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Decision-Making Processes III

- Menthe argues that decision-making involves modelling:
  - “The core of this model is constituted by the situational analysis; all the aspects that are important, for the student, to pass the judgement converge into [the situational analysis]. **The Situational analysis, is the *picture* or, in other words the map, which the student constructs of the issue on which judgement must be made.** This involves ideally all relevant scientific ... and personal ... factors, available to the student. ... The person assigns a value to ... the different action options with respect to personal values, [the person] weighs these and arrives in this way to the, for him/her, appropriate solution” (Menthe, 2006, p. 33; my emphasis and translation).

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Designing the vertical structure

- Remember the dilemma: how do we avoid
  - the Scylla of radical student autonomy
  - the Charybdis of no student autonomy
- One possible answer: the *Anthropological Theory of the didactics*

The Anthropological Theory of Didactics

- The *Anthropological Theory of Didactics* (See e.g. Chevallard, 1991):
  - Learning can be spelled out in the acquisition/construction/ development of continually more complex *praxeologies*
  - The notion of praxeology: we “can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand” (Chevallard, 2006, p. 23)
  - Praxeologies (and knowledge in general) are intersubjectively shared by, and idiosyncratic to, groupings such as social classes.
  - To teach means that official praxeologies or knowings of the scientific/mathematical community must be *transposed* into the classroom context (Chevallard, 1985)

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ATD and the phenomena and task of education

- The key phenomena of education:
  - Facilitate that students acquire and activate new and more demanding praxeologies by being posed problems that are *specifically designed* for the educational context
- The task of the educator:
  - Relevant bodies of knowledge must be transposed in such a way as to afford meaningful and sensible trajectories of situations that call for the application of increasingly complex praxeologies.

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Initial concerns

- The “paradox of indoctrination”:
  - “if indoctrination means instilling beliefs without reasons, and if children lack the rational capacity to evaluate reasons, how can that capacity be cultivated without indoctrination?” (Hanks, 2008, p. 193)
- This problem is important
  - If knowledge and praxeologies are *transposed* and *represented* to students how will they acquire the traits of critical citizenry?
- Chevallard is very aware of this and he accounts for how we, within ATD, can secure that students are “finding thigs *out*” instead of merely finding things by visiting “monuments” (Chevallard, 2006, p. 29).
- SRC is his answer!

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Study & Research Courses

- ***A design approach for modelling activities***
- *Research* - the modus of student participation (finding things out)
- *Course* - like a golf course (along a trajectory)

- The course is “determined essentially by the will to bring an answer, *A*, to some *generating question, Q*” (Chevallard, 2006, p. 28).
- The didactical design approach in SRC is accordingly to articulate a generating question for a given subject area and foresee, through a priori analysis, a “theoretical trajectory” (Winsløw, 2009) of this course.

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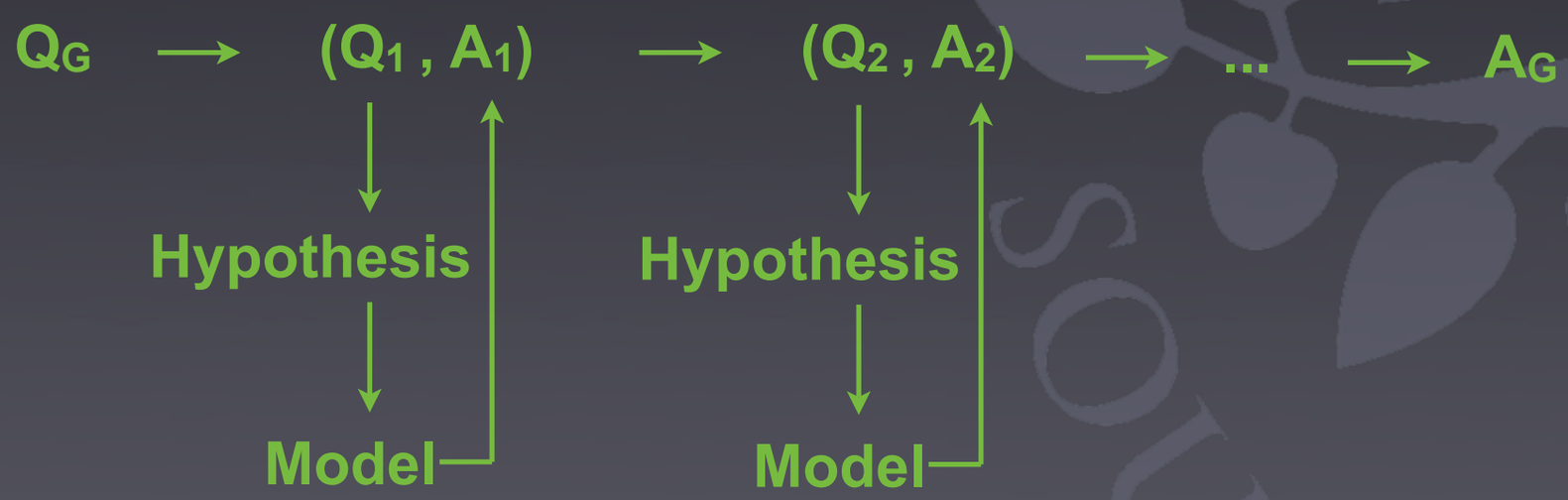
The structural role of *Q*

- Baquero, Bosch, and Gascón (2007): SRC allows students to research a given problem that,
  - Generates a series of increasingly more difficult and complex sub-problems, and
  - Itself can only be comprehensively solved through the interaction with its sub-problems - i.e. the process of solving the generative problem is scaffolded by the sub-problems it itself generates.



After Mortensen (2009) and Barquero (2008)

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Example of SRC*

- Generating Question $\mathbf{Q_G}$: Is population overgrowth a problem?
- A possible trajectory:
  - How is the population at time t_n related to the population at time t_{n-1}?
    - Discrete model: $x_{n+1} = Mx_n = M^{n+1}x_o$ (r_n constant)
    - Discrete logistic model: $r_{n+1} = f(x_n)$ (r_n linear)
    - General functional model (ord. diff. equation) $x'=f(x)$

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* See e.g. Barquero, 2009.

Final remarks

- I have tried to vindicate the theoretical connection between decision-making, interdisciplinarity and modelling competencies.

- This could be valuable for material design considerations:
  - Placing students in situations where they need to make a socio-mathematical decision is well suited for an interdisciplinary modelling process

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Final remarks

- Choosing a **sensible** problem issue on which the decision must be made provides
  - A way to structure the conceptual field in which the processes of reorganisation happen
  - Projected trajectories of modelling processes through that field
- As a slogan:
  - Well analysed and explicit aims for the horizontal linking can in themselves guide and structure the vertical processes

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