University of Education Schwäbisch Gmünd

Supporting Mathematical Literacy

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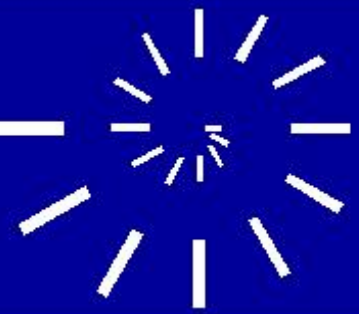
Teaching approaches from the Cross-Curricular European Project **ScienceMath**
here: German Group

Prof. Dr. Astrid Beckmann, coordinator of ScienceMath

Beijing August 2008, China

Prof. Dr. Astrid Beckmann, Europe, Germany



1 Mathematical literacy

2 The ScienceMath project

- **Example 1: concept of function**
- **Example 2: center of mass**
- **Example 3: Fernat meets Pythagoras**
- **Example 4: nutrition cycle**

3 Conclusion and Perspectives

1 Mathematical literacy

→Basic objective of modern maths instruction

Definition (OECD* 2006):

„Mathematical literacy is an individual´s capacity to identify and understand the role that mathematics plays in the world, to make wellfounded judgements and to use and engage with mathematics in ways that meet the needs of that individual´s life as a constructive, concerned and reflective citicen.“

*Organisation for Economic Co-operation and Development

1 Mathematical literacy

General definition (Jablonka 2003):
Subdividing mathematical literacy into 5 fields:

„*Mathematical literacy for*
- developing Human Capital
- Cultural Identity
- Social Change
- Environmental Awareness
- Evaluating Mathematics“

European Curricula:

→ *Improtant role:*
Use of mathematics for the creation of models

1 Mathematical literacy

To conclude:

Core of mathematical literacy:

- ability to apply mathematical knowledge to various and context-related problems in a functional, flexible and practical way

This includes:

- „**formal knowledge**“ Use simple structures and methods (calculations)

- „**applicable knowlegde**“ Use mathematical concepts and structures, relation between them, to deal with them in unknown situations

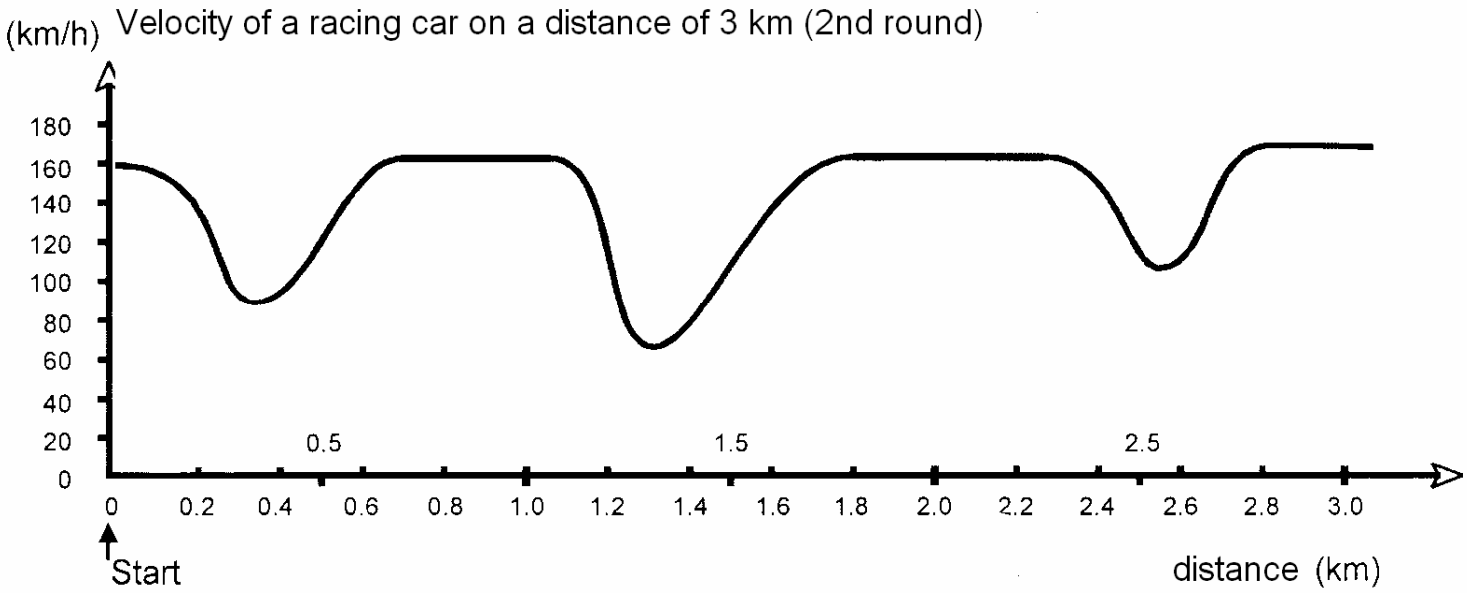
Deficits

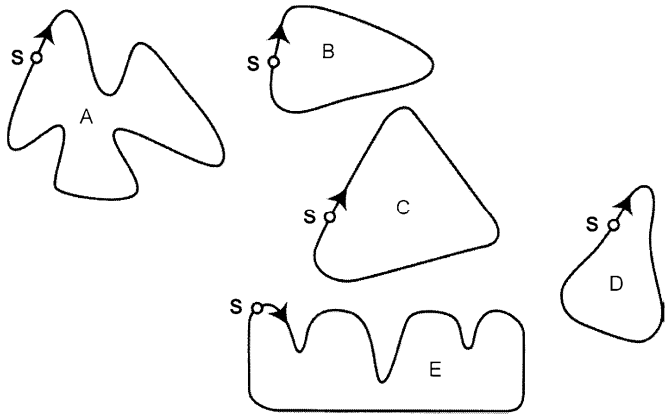
1 Mathematical literacy

E.g. PISA: German students failed when asked to solve problems or master tasks that were unfamiliar situations

→ Deficits in problem solving competencies
in the field of applied mathematics
(no abilities in interpreting graphs or modeling activities etc.)

(PISA 2003):


(km/h)
Velocity of a racing car on a distance of 3 km (2nd round)
180
160
140
120
100
80
60
40
20
0
0.5
1.5
2.5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
Start
distance (km)


S
A
S
B
S
C
S
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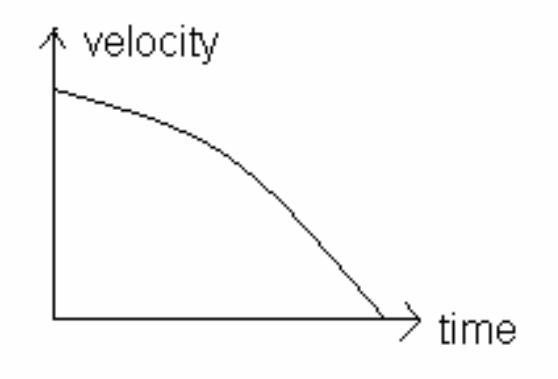
S
E

(own investigation)

The picture shows the movement of a car.
What kind of traffic situation is the car in?



Typical student´s answer: „The car is rolling downhill“.

1 Mathematical literacy

Consequences for teaching courses:

- Connect conventional mathematics and authentic experiences
- Present Problems in terms of realisitic situations
- Use cross-curricular approaches
- Stimulate (all kind of) modelling activities
- Initiate wide experiences with a mathematics concept

2 The ScienceMath project

A European Cooperation project between universities and schools of

Germany 

Denmark 

Finnland 

Slovenia 

Supported by the
European Commission

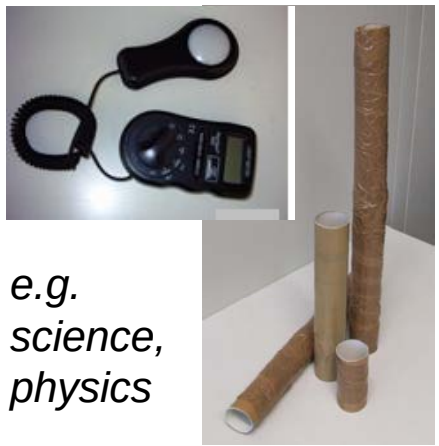
The basis of the project is an interdisciplinary approach with sciences especially with Physics.
The pupils shall experience Mathematics in an appropriate interesting and important way
by the means of extra-mathematical references;
learning in interrelations shall contribute to an intuitive mathematic understanding.
With the aid of scientific contexts and methods the gap between formal mathematics
and authentic experience shall be closed and on the other hand
the variety of mathematic items shall be experienced

2 The ScienceMath project

Approaches:

- realistic situations

- cross-curricular



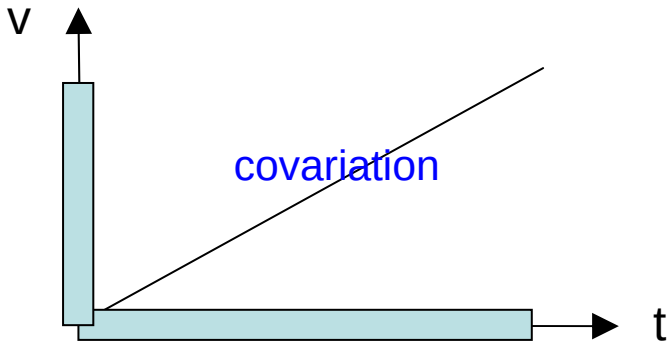
e.g. science, physics

- authentic experiences

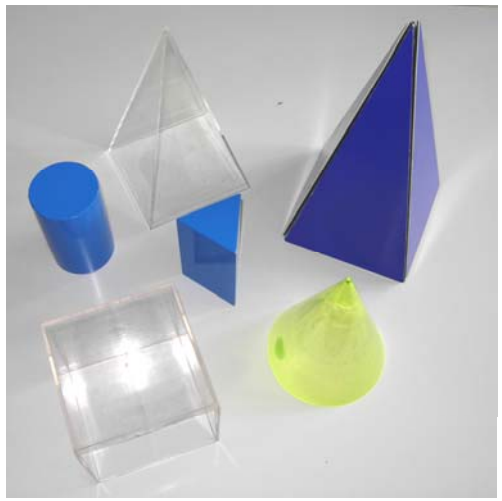
2 The ScienceMath project

Approaches:

- modelling activities

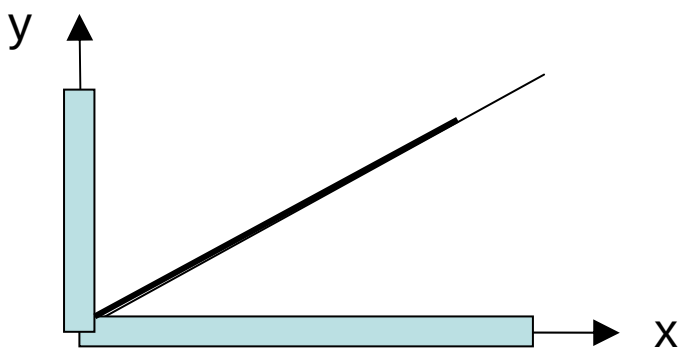


- wide experiences with a term



Example 1: Concept of Function

From a test: *Describe the dependence between x and y*



Shortened concept understanding:

No idea of covariation

Student´s answer: „The line devides the system of coordinates“

Example 1: Concept of Function

Initiating authentic and wide experiences with a mathematical term
(here: concept of function/ the aspect of covariation)
Using cross-curricular approaches

Experiment:
Material: Car with drive, measuring cord (at least 2 m), stop watches
Performance: The car moves with constant velocity.
Measurement: Timing for certain distances of the car

As time goes on, the distance of the car increases.

Example 1: Concept of Function

Presenting problems in terms of realistic situations and stimulate modelling activities

Experiment:

Impulse: worksheet with the following task:

Imagine you are going into a tunnel and you cannot see the end.
How does the brightness (intensity of light) changes
while going into it?

Material: measuring instrument for brightness measurement (lux-meter), cardboard tubes of different length and same diameter (tunnels), window with day-light

Performance: The card-board tubes are held with one side at the window. At the other side the sensor of the lux-meter is fixed.

Measurement: The brightness is directly shown at the display.

Example 1: Concept of Function

Present problems in terms of realistic situations and stimulate modelling activities

From a student´s report (14 years old):

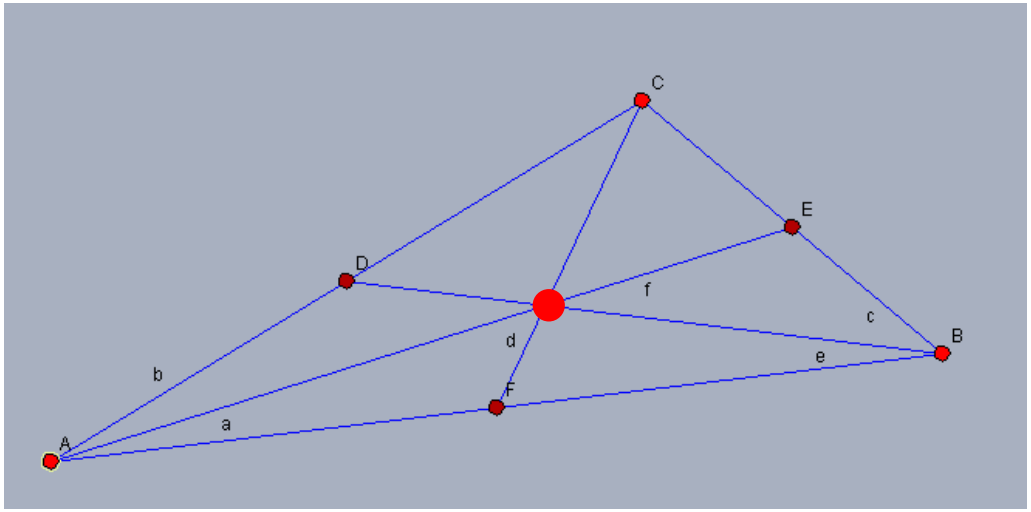
„We called the second project ´light and tunnel´.
The further the car drove into the tunnel the darker it got.
The first tube had a length of 9.7 cm.
When we held it to the window, the intensity of the light was approximately 36 lux.
When we held a 30 cm tube to the window, we only measured 0.1 lux.
In a graph we were able to establish exactly how much light there was at the entrance to the tunnel and how much there was left at the end of the tunnel.“

Example 2: Center of mass

Connect conventional mathematics and authentic experiences,
Initiating authentic and wide experiences with a mathematical term (here: intersection of medians),
Using cross-curricular approaches

The idea: Experiencing the intersection of the medians in a triangle as a special case of the concept of the center of math

A task of traditionally math lesson:
Draw a triangle.
Construct the medians.



The medians intersect in one point.

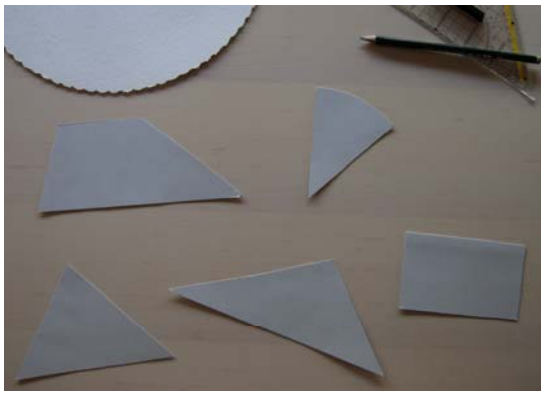
Example 2: Center of mass

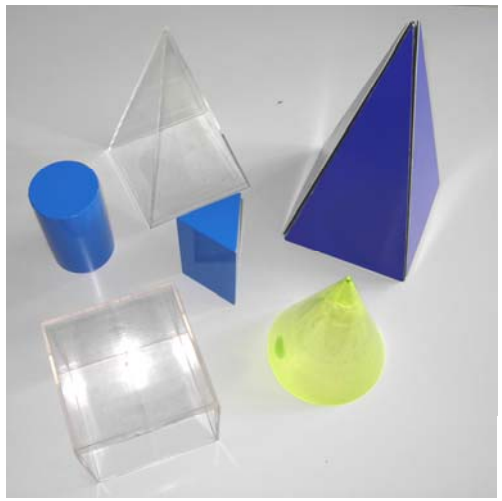
Connect conventional mathematics and authentic experiences,
Initiating authentic and wide experiences with a mathematical term (here: intersection od medians),
Using cross-curricular approaches

The intersection of the medians is the center of mass.

↓

This motivates: science experiments and more experiences (triangle, rectangles,… solids etc.)



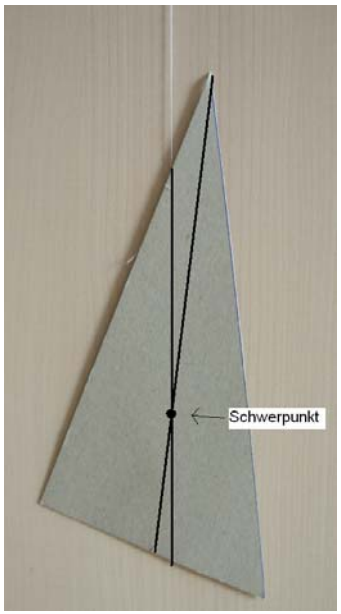


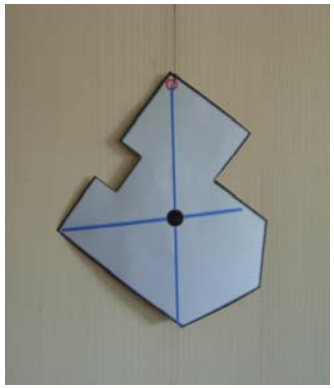
Example 2: Center of mass

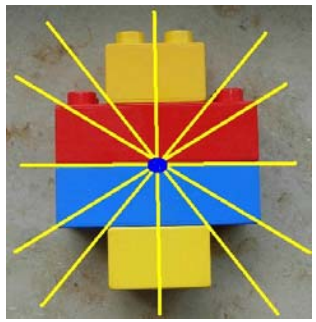
Initiating authentic and wide experiences with a mathematical term

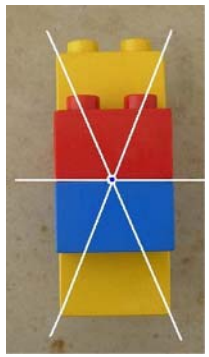
Experiences:

Hang method

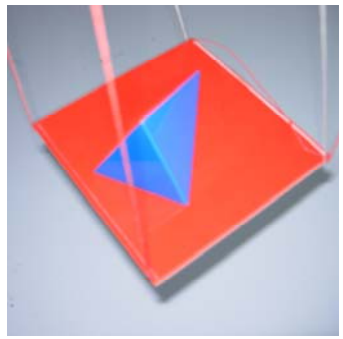


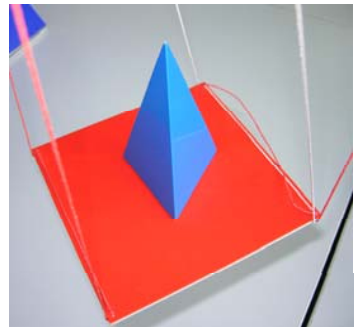






Weight method





Example 2: Center of mass

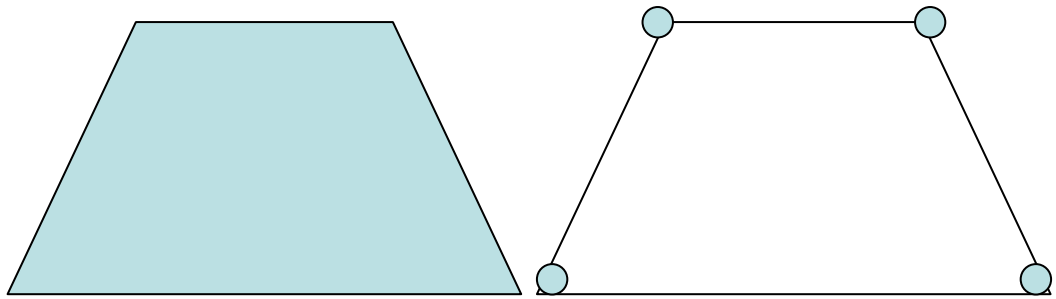
Presenting problems in terms of realistic situations and stimulate modelling activities

Center of mass in a triangle: $\vec{s} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$

Center of mass in a quadrilateral: $\vec{s} = \frac{1}{4}(\vec{a} + \vec{b} + \vec{c} + \vec{d})$ *valid in general?*

Center of mass in general:

$$\vec{s} = \frac{1}{m_1 + \ldots + m_k} \sum_{i=1}^{k} m_i \vec{x_i}$$



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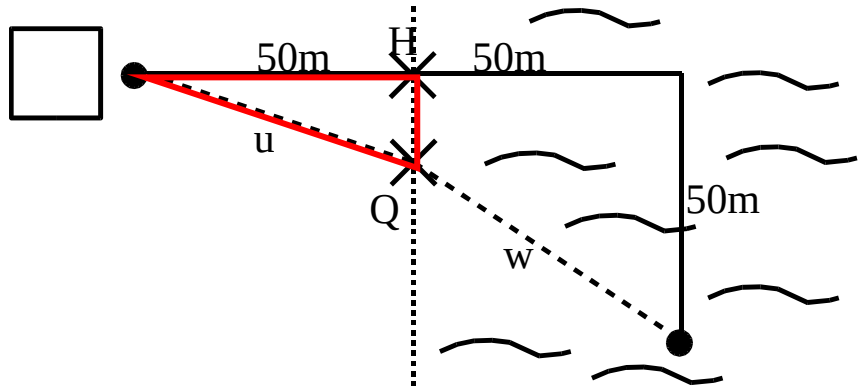
Example 3: Fermat meets Pythagoras*

Presenting problems in terms of realistic situations and stimulate modelling activities

The Idea: Experience the connection between the Pythagoras´ Theorem and the Principle of Fermat through the discussion of extremum problems and science experiments

Lifeguard Mitch is standing in front of his tower
when he views a person in distress in the water.
The direct way to the water is 50 m.
From there it is another 50 m straight on
and then 50 m south to get to the person in need.
Mitch knows that he needs 7m/s ashore
and only 2m/s in the water.
In order to get to the person as quick as possible
he starts off by running straight along the shore
to a point Q from where he swims directly to the person.
On the shore he covers a distance of u meters
and w meters in the water.



Determine the distance HQ so, that the lifegard is quickest.

*idea: Thilo Höfer

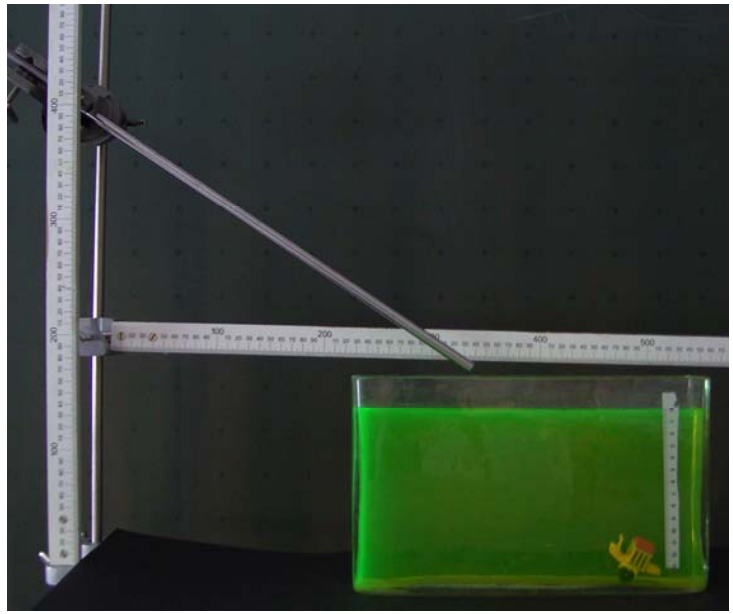
Example 3: Fermat meets Pythagoras

Using cross-curricular approaches,
Presenting problems in terms of realistic situations and stimulate modelling activities

Light does not always travel at the same speed.
The speed of light in air is 300.000 km/s
whereas the speed of light in glass
and water is only about 200.000 km/s
and 225.000 km/s, respectively.

As well as that, light always acts as a perfect lifeguard.
This means, a beam of light always chooses the shortest way possible to get from a point A to a point B.

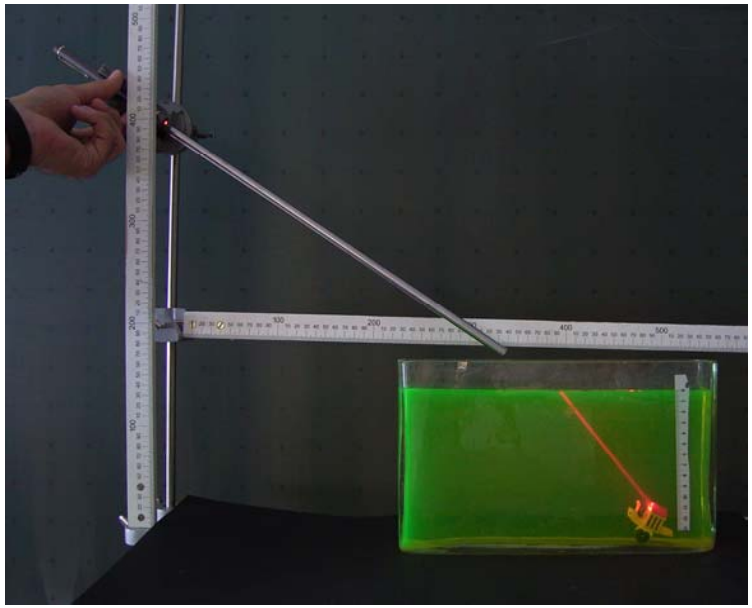
This behaviour of light is called Fermat's Principle
-named after Pierre de Fermat (1608-1665)
-who was the first to discover this.



Where does point Q have to be in order for the light to travel the fastest way from L to Q and from Q to S?

Example 3: Fermat meets Pythagoras*

Using cross-curricular approaches,
Presenting problems in terms of realistic situations and stimulate modelling activities



$$t_u = (\sqrt{(50^2 + x^2)}m) : (7m/s)$$

$$t_w = (\sqrt{(50^2 + (50-x)^2)}m) : (2m/s)$$

$$t_1 = \sqrt{26^2 + x^2} \,/\, 30.000.000.000 \text{ s (in air)}$$

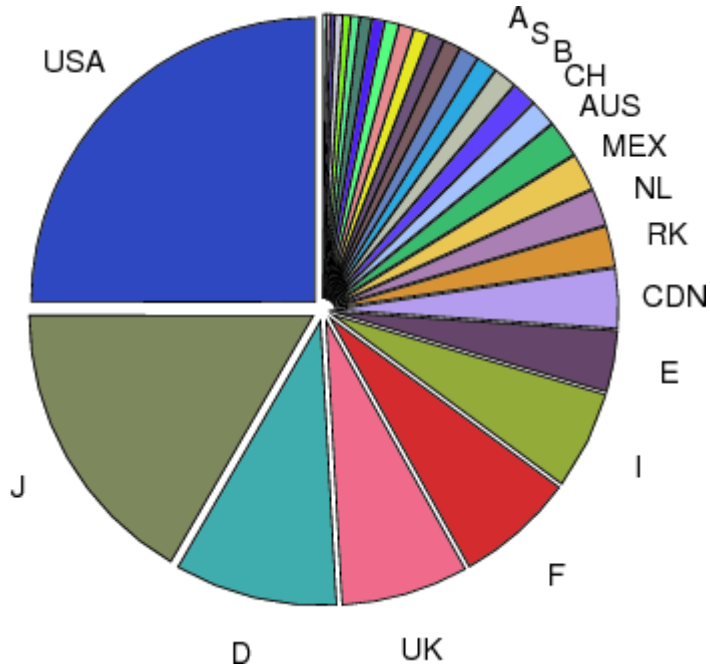
$$t_2 = \sqrt{11^2 + (50-x)^2} \,/\, 22.500.000.000 \text{ s (in water)}$$

Example 4: Circle Diagramms and Nutrition Cycle*

A difficult but important theme in mathematics lessons:
Diagramms, especially circle diagramms

E.g.: OECD Budget 2007: 341.8 million Euro

USA	85.1 million
Japan	54.7 million
Germany	31.1 million
Australia	7.3 million
Iceland	0.6 million



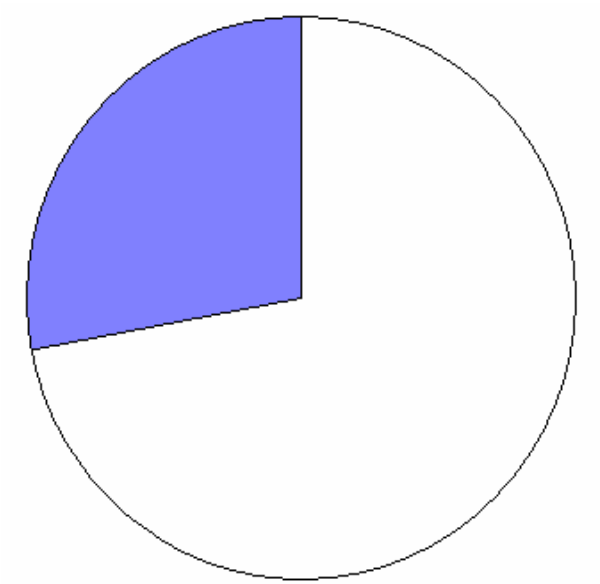
*idea: Annika Grube

Example 4: Circle Diagramm and Nutrition Cycle*

World Population: 6.7 billion

China	1325 million	20 %
India	1132 million	17 %
USA	302 million	4.6 %
…		
Nigeria	144 million	2.2 %
Germany	82 million	1.2 %

20% of 360° = 72°



*idea: Annika Grube

Example 4: Circle Diagramm and Nutrition Cycle*

Using cross-curricular approaches,
Presenting problems in terms of realistic situations and stimulate modelling activities

The Idea: Learning percantage calculation in the context of circle graph

Biology background

Nutrition cycle:
Recommendation of the German Society of Nutrition

*idea: Annika Grube

Task: Note all what you ate the day (in gramme)

	Cereal	Vegetables	Fruits	Milk products	Meat, Eggs	Sweets
Breakfast						
School						
Noon						
Afternoon						
Evening						
Total						

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Task: Calculate the Total and calculate the percentage of each category						
	Cereal	Vegetables	Fruits	Milk products	Meat, Eggs	Sweets
%	31 %		15 %			
Corresponding angle	111,6°		54°			

Using cross-curricular approaches,
Presenting problems
in terms of realistic situations
and stimulate modelling activities

3 Conclusions and Perspectives

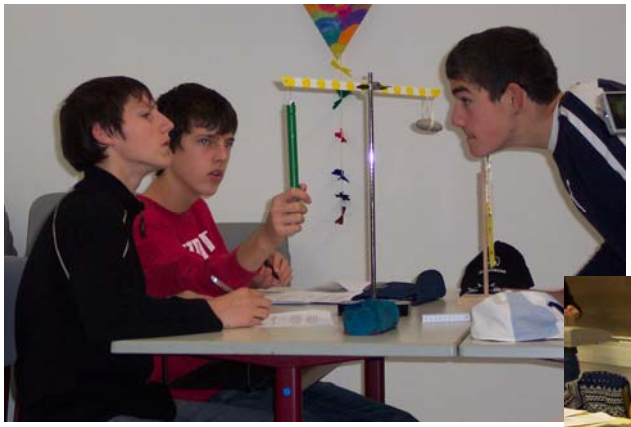
Four Examples….

- Using cross-curricular approaches
- Initiating authentic and wide experiences with a mathematical term
  (here: concept of function/ the aspect of covariation
  and: intersection of medians/ the aspect of center of mass)
- Presenting problems in terms of realistic situations and stimulate modelling activities

→ Supporting mathematical literacy ?

3 Conclusions and Perspectives

ScienceMath Project:
Testing of the sequences/ modules



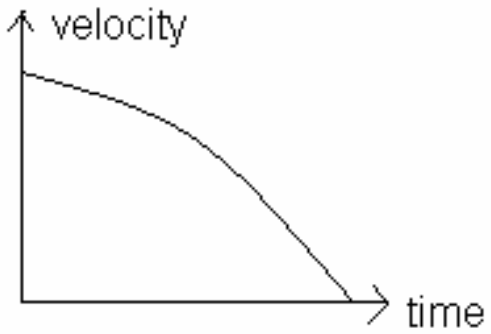
3 Conclusions and Perspectives

→ Supporting mathematical literacy ?

Some steps into the right direction…

Concept of function (result from a post-test):

The picture shows the movement of a car.
What kind of traffic situation is the car in?



„The car is slowing down.“

„It is stopping at the crossroads“

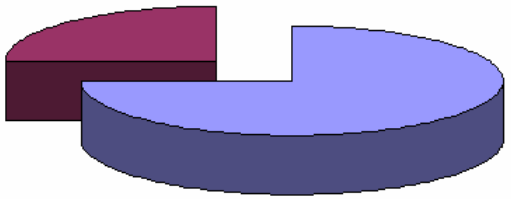
„It has to brake because of a barrier.“

Some steps into the right direction…

Nutrition Cycle

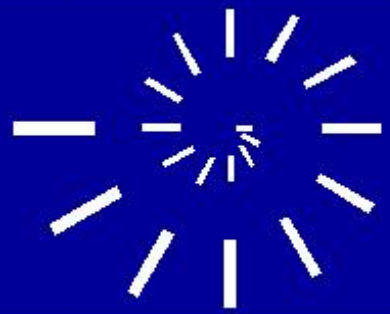
Great motivation leaded the work:
Mathematics allow the comparison of the own nutrition behaviour with the recommended circle.
and a discussion upon the backdrop of health.

e.g. boys



girls

Meat or Cereals



More examples and results from testing:

www.sciencemath.ph-gmuend.de